L 1115

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2006.

First Semester

Civil Engineering

MA 1101 — MATHEMATICS – I

(Common to all branches of Engineering and Technology)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Let λ be an eigen value of a non-singular matrix A with eigen vector x. Show that $\frac{1}{\lambda}$ is an eigen value of A^{-1} with eigen vector x.

2. Classify the quadratic forms $x_1^2 + x_3^2$ and $x_1^2 - x_2^2$.

3. Find the values of K, if the lines $\frac{x-2}{3} = \frac{y-1}{2} = \frac{z-3}{K}$ and $\frac{x-3}{K} = \frac{y-2}{3} = \frac{z-4}{5}$ are coplanar.

4. Find the equation of the tangent plane at the point (1, –1, 2) to the sphere $x^2 + y^2 + z^2 - 2x + 4y + 6z - 12 = 0$.

5. The curvature of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $(a\cos\theta, b\sin\theta)$ is $\frac{ab}{(a^2 \sin^2\theta + b^2\cos^2\theta)^{\frac{3}{2}}}$. Show that the eccentricity of the ellipse is $\frac{1}{\sqrt{2}}$, if the center of curvature at one end of the minor axis lies at the other end.

6. Find the envelope of the family of lines $\frac{x}{t} + yt = 2C$, t being the parameter.

7. If $\sin zy = \cos zx$, compute $\frac{\partial z}{\partial x}$ when $z = \pi$, $x = \frac{1}{3}$ and $y = \frac{1}{6}$.

8. If $u = f(y-z, z-x, x-y)$, find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.

9. Find the particular integral of the differential equation $(D^2+1)y = x^3$.

10. Transform the equation $xy'' + y' + 1 = 0$ into linear equation with constant coefficients.

PART B — (5 × 16 = 80 marks)

11. (i) Find the matrix A, whose eigen values are 2, 3 and 6, and the eigen vectors are $\{1,0,-1\}^T$, $\{1,1,1\}^T$ and $\{1,-2,1\}^T$. (8)

(ii) If $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, find A^{-1} and A^4 using Cayley-Hamilton theorem. (8)

12. (a) (i) Find the equation of the plane through the points (1, –2, 2) and (–3, 1, –2) and perpendicular to the plane $2x + y - z + 6 = 0$. (6)

(ii) Show that the lines $x + y + z - 3 = 0 = 2x + 3y + 4z - 5$ and $4x - y + 5z - 7 = 0 = 2x - 5y - z - 3$ are coplanar and find the equation of the plane in which they lie. (10)

Or

(b) (i) Find the area of the circle in which the sphere $x^2 + y^2 + z^2 + 12x - 2y - 6z + 30 = 0$ is cut by the plane $x - y + 2z + 5 = 0$. (5)

(ii) Find the length and equation of the shortest distance between the lines $\frac{x+1}{2} = \frac{y+1}{3} = \frac{z+1}{4}$ and $\frac{x+1}{3} = \frac{y}{4} = \frac{z}{5}$. (11)

13. (a) (i) Find the center of curvature of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(a\sec\theta, b\tan\theta)$. (8)

(ii) Find the equation of the evolute of the curve $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$. (8)

Or

(b) (i) If ρ_1 and ρ_2 be the radii of curvature at the ends of any chord of the cardioid $r = a(1+\cos\theta)$, that passes through the pole, prove that $\rho_1^2 + \rho_2^2 = \frac{16a^2}{9}$. (8)

(ii) Find the envelope of the straight line $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are parameters that are connected by the relation $a + b = c$. (8)

14. (a) (i) If $u = f(x, y)$ and $x = r\cos\theta$, $y = r\sin\theta$, find $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$. (8)

(ii) Find the length of the shortest line from the point $\left(0, 0, \frac{25}{9}\right)$ to the surface $z = xy$. (8)

Or

(b) (i) Expand $x^2 y + \sin y + e^x$ in powers of $(x-1)$ and $(y-\pi)$ through quadratic forms. (8)

(ii) Find the maximum and minimum values of $2(x^2 - y^2) - x^4 + y^4$. (8)

15. (a) (i) Solve $(D^2 + 5D + 4)y = e^{-x}\sin 2x + x^2 + 1$ where $D = \frac{d}{dx}$. (8)

(ii) Solve the equation $x^2 \frac{d^2 y}{dx^2} - x\frac{dy}{dx} + y = x\log x$, by the method of variation of parameters. (8)

Or

(b) (i) Solve $(D+4)x + 3y = t$ and $2x + (D+5)y = e^{2t}$ where $D = \frac{d}{dt}$. (8)

(ii) Solve $x^2 \frac{d^2 y}{dx^2} - x\frac{dy}{dx} + y = x\log x$. (8)

———————————