M 2058

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2006.

Third Semester

Civil Engineering

MA 1201 — MATHEMATICS — III

(Common to All branches (EXCEPT Bio-medical Engg.) and common to B.E. (P.T.) Second Semester R 2005 Civil, Computer Science, Electrical and Electronics, Electronics and Communication and Mechanical Engg.)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Form the PDE by eliminating the arbitrary function from $\phi\left[z^2 - xy, \frac{x}{z}\right] = 0$.

2. Find the singular integral of the partial differential equation $z = px + qy + p^2 - q^2$.

3. Find the Fourier constants bn for $x \sin x$ in $(-\pi, \pi)$.

4. State Parseval's identity for the half-range cosine expansion of $f(x)$ in $(0,1)$.

5. What are the possible solutions of one dimensional wave equation?

6. In steady state conditions derive the solution of one dimensional heat flow equation.

7. Find the Fourier sine transform of $f(x) = e^{-x}$.

8. Prove that $F\left[f(ax)\right] = \frac{1}{a} F\left(\frac{s}{a}\right)$, $a > 0$.

9. State and prove initial value theorem in z–transform.

10. Find the z–transform of $(n+1)(n+2)$.

PART B — ($5 \times 16 = 80$ marks)

15.

11. (i) Using convolution theorem evaluate inverse z–transform of $\left[\frac{z^2}{(z-1)(z-3)}\right]$. (6)

(ii) Using z–transform solve $y(n)+3y(n-1)-4y(n-2)=0$, $n \geq 2$ given that $y(0)=3$ and $y(1)=-2$. (10)

12. (a) (i) Solve $(y^2+z^2)p - xyq + xz = 0$. (8)

(ii) Solve $(D^2 - 6DD' + 5D'^2)z = e^x \sinh y + xy$. (8)

Or

(b) (i) Solve $p(1-q^2) = q(1-z)$. (8)

(ii) Solve $(D^2 - 4DD' + 4D'^2)z = e^{2x+y}$. (8)

13. (a) Find the Fourier series for $f(x) = |\cos x|$ in the interval $(-\pi, \pi)$. (16)

Or

(b) (i) Find the half range sine series for $f(x) = x(\pi - x)$ in the interval $(0, \pi)$ and deduce that $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \cdots + \infty$. (8)

(ii) Find the Fourier series upto second harmonic for the following data :

x :	0	1	2	3	4	5
y :	9	18	24	28	26	20

(8)

14. (a) A string of length l has its ends $x = 0, x = l$ fixed. The point where $x = l/3$ is drawn aside a small distance h, the displacement $y(x, t)$ satisfies $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$. Find $y(x, t)$ at any time t. (16)

Or

(b) An infinitely long rectangular plate with insulated surface is 10 cm wide, and the two long edges and one short edge are kept at 0°C temperature, while the other short edge $x = 0$ is kept at temperature given by

$$u = \begin{cases} 20y & \text{for } 0 \leq y \leq 5 \\ 20(10-y) & \text{for } 5 \leq y \leq 10. \end{cases}$$

Find the steady state temperature distribution in the plate. (16)

15. (a) (i) Find the Fourier transform of

$$f(x) = \begin{cases} 1 - x^2 & \text{in } |x| \le 1 \\ 0 & \text{in } |x| > 1 \end{cases}$$ and hence deduce that

$$\int_0^\infty \frac{\sin S - S\cos S}{S^3} \cos\frac{S}{2}\, dS .$$ (8)

(ii) Find the Fourier cosine transform of $e^{-a^2x^2}$. (8)

Or

(b) (i) Find the Fourier transform of $f(x)$ if

$$f(x) = \begin{cases} 1 - |x|, & |x| < 1 \\ 0 & |x| > 1 \end{cases}$$ and hence evaluate $\int_0^\infty \frac{\sin^4 t}{t^4}\, dt$. (8)

(ii) Using convolution theorem, evaluate $\int_0^\infty \frac{dx}{(x^2 + a^2)(x^2 + b^2)}$. (8)
