B 470

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2006.

Fourth Semester

Electronics and Communication Engineering

MA 034 — RANDOM PROCESSES

Time : Three hours Maximum : 100 marks

Statistical Tables are Permitted.

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Distinguish between unconditional and conditional probabilities.

2. Define a Poisson variate.

3. If two random variables X and Y have probability density function (PDF) $f(x, y) = k\, e^{-(2x+y)}$ for $x, y > 0$, evaluate 'k'.

4. Write a note on 'central limit theorems'.

5. When is a random process said to be ergodic?

6. What is a Markov process?

7. Give an example of stationary random process and justify your claim.

8. Define cross correlation function.

9. When is a system said to be linear?

10. Define autocorrelation function.

PART B — (5 × 16 = 80 marks)

11. (i) Distinguish between spectral distribution function '$F(.)$, say' and spectral density function '$f(.)$, say' based on the properties of the covariance stationary process $\{X(t); \infty < t < \infty\}$. For such a process, if the auto correlation function is $R_{XX}(\tau) = \sigma^2 \ e^{-\alpha|\tau|}$, find the power spectral density. (8)

(ii) Distinguish between a single $x(t)$ = input, $y(t)$ = output of a linear system and a linear time invariant system with impulse response function $h(.)$. If the process $\{x(t)\}$ is wide sense stationary, obtain the autocorrelation function of $x(t)$ and cross correlation function of $x(t)$ and $y(t)$ in terms of the impulse response $h(.)$. (8)

12. (a) (i) What are independent events? Two weak students attempt to write a program. Their chances of writing the program successfully is 1/8 and 1/12 and the chance of making a common error is 1/10001. Find the chance that the program is correctly written. (8)

(ii) Describe gamma distribution. Obtain its moment generating function (MGF). Hence or otherwise compute the first four moments. (8)

Or

(b) (i) Suppose the duration 'X' in minutes of long distance calls from your home, follows exponential law with PDF $f(x) = (1/5)e^{-(x/5)}$ for $x > 0$, 0 otherwise find $P[X > 5], P[3 \le X \le 6]$, mean of X and variance of X. (8)

(ii) Describe negative binomial distribution $X \sim NB(k, p)$ where X = number of failures preceding the k^{th} success in a sequence of Bernoulli trials and p = probability success. Obtain the MGF of X, mean and variance of X. (8)

13. (a) (i) The joint probability mass function (PMF) of random variables X and Y is $p(x, y) = \dfrac{e^{-\lambda} \ \lambda^{x} \ p^{x} \ q^{x-y}}{y! \qquad (x-y)!}$; $y = 0,1,2,\ldots x$ and $x = 0,1,2\ldots$ where $\lambda > 0, 0 \le p \le 1, \ p + q = 1$ are constants. Find the marginal and conditional distributions. (8)

(ii) Let the number X be selected from among the set of integers $\{1,2,3,4\}$ and the number Y be chosen from among those at least as X. Obtain the joint PMF of (X, Y). Hence prove that covariance $(X, Y) = 5/8$. (8)

Or

(b) (i) Two dimensional random variable (X, Y) has the joint PDF $f(x, y) = 8xy, 0 < x < y < 1 ; 0$ otherwise Find (1) marginal and conditional distributions, and (2) whether X and Y are independent. (8)

(ii) Two random variables X and Y are defined as $Y = 4X + 9$. Find the coefficient of correlation between X and Y. (8)

14. (a) Define a Markov Chain (MC) and its one step transition probability matrix (tpm). In a hypothetical market there are only two brands A and B. A customer buys brand A with probability 0.7 if his last purchase was A and buys brand B with probability 0.4 if his last purchase was B. Assuming MC model, obtain (i) one step tpm 'P say', (ii) n step tpm P^n, and (iii) the stationary distribution. Hence highlight the proportion of customers who would buy brand A and brand B in the long run.

Or

(b) Write a short note on each of the following processes : $(4 \times 4 = 16)$

(i) Binomial process,

(ii) Sine–Wave process

(iii) Random walk process and

(iv) Ergodic process.

15. (a) (i) Show that the random process $\{X(t) = A \cos(wt + \theta)\}$ is wide sense stationary where A and w are constants and θ is uniformly distributed on the interval $(0, 2\pi)$. (8)

(ii) Determine whether or not the process $\{X(t) = A \sin(t) + B \cos(t)\}$ is ergodic, if A and B are normally distributed random variables with zero means and unit variances. (8)

Or