L 1117

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2006.

Second Semester

Civil Engineering

MA 1151 — MATHEMATICS — II

(Common to all Branches of Engineering and Technology)

(Regulations 2004)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Evaluate $\int_4^3\int_1^2 (x+y)^{-2}\,dx\,dy$.

2. Change the integration $\iiint f(x,y,z)\,dx\,dy\,dz$ into spherical polar co-ordinates.

3. Prove that $\nabla(r^n)=n\,r^{n-2}\,\vec{r}$.

4. If $\vec{V}=(x+3y)i+(y-2z)j+(x+\lambda z)k$ is Solenoidal find λ.

5. Show that an analytic function with constant real part is constant.

6. Find the fixed points of $\omega=(3z-4)/(z-1)$.

7. Evaluate $\int_c dz/(z-3)^2$ where C is the circle $|z|=1$.

8. Find the Laurent's series expansion of $f(z)=e^{2z}/(z-1)^3$ about $z = 1$.

9. Find $L[t\sin 2t]$.

10. Find $L^{-1}\left[\dfrac{1}{(s-3)^2}\right]$.

PART B — (5 × 16 = 80 marks)

11. (i) Change the order of integration and then integrate $\int_0^1\int_{x^2}^{2-x} xy\,dy\,dx$. (8)

(ii) By transforming into cylindrical polar co-ordinates evaluate $\iiint (x^2+y^2+z^2)\,dx\,dy\,dz$ taken over the region of space defined by $x^2+y^2\le 1$ and $0\le z\le 1$. (8)

12. (a) (i) Find the work done when a force $F = \left(x^2 - y^2 + x\right)i - (2xy + y)j$ moves a particle from the origin to the point (1, 1) along $y = x^2$. (8)

(ii) Evaluate $\int_S \overline{F}.ds$ where $\overline{F} = 4x\,i - 2y^2 j + z^2 k$ and S is closed surface bounding the cylinder $x^2 + y^2 = 4,\ z = 0,\ z = 3$. (8)

Or

(b) (i) Find the values of a and b so that the surfaces $ax^3 - by^2 z = (a+3)x^2$ and $4x^2 y - z^3 = 11$ may cut orthogonally at (2, –1, –3). (8)

(ii) Verify Gauss divergence theorem for the function $\overline{F} = yi + xj + z^2 k$ over the cylindrical region bounded by $x^2 + y^2 = 9, z = 0$ and $z = 2$. (8)

13. (a) (i) If $f(z) = u + iv$ is regular function then prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left[|f(z)|^2\right] = 4\,|f'(z)|^2$$ (8)

(ii) Find the analytic function whose real part is $\sin 2x/(\cosh 2y - \cos 2x)$. (8)

Or

(b) (i) Find the Bilinear transformation that maps the points $Z = -1, 0, 1$ in the z – plane on to the points $\omega = 0, i, 3i$ in the ω – plane. (8)

(ii) Show that $V = e^{2x}\left(y\cos 2y + x\sin 2y\right)$ is harmonic and find the corresponding analytic function $f(z) = u + iv$. (8)

14. (a) (i) Evaluate $\int_c (z-2)/z\,(z-1)\,dz$ where C is $|z| = 3$. (8)

(ii) Find the Laurent's Series expansion of $(z-1)/(z+2)(z+3)$ valid in the region $2 < |z| < 3$. (8)

Or

(b) (i) Evaluate $\int_0^{2\pi} \cos 2\theta/(5 - 4\cos\theta)\,d\theta$ using contour integration. (8)

(ii) Evaluate $\int_0^{\infty} dx/\left(x^2 + a^2\right)\left(x^2 + b^2\right)$ $a > 0; b > 0$ using contour integration. (8)

15. (a) (i) Find the Laplace Transform of $e^{3t}\,[t \cos 2t]$ and $\dfrac{1-e^{-2t}}{t}$. (8)

(ii) Solve the differential equation, using Laplace transform

$y'' + 4y' + 4y = e^{-t}$ given that $y(0) = 0$ and $y'(0) = 0$. (8)

Or

(b) (i) Find the Laplace transform of a triangular wave function

$$f(t) = t \qquad 0 < t < a$$

$$= 2a - t \qquad a < t < 2a \text{ where } f(t + 2a) = f(t).$$ (8)

(ii) Using Convolution Theorem find the Laplace inverse of

$(s+2)/(s^2+4s+13)^2$ (8)
