

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2006.

Fourth Semester

Automobile Engineering

MA 1253 — PROBABILITY AND STATISTICS

(Regulation 2004)

(Common to Mechanical Engineering, Production Engineering, Information Technology, Textile Technology, Textile Technology (Fashion Technology), Textile Technology (Textile-Chemistry) and Common to B.E. (Part-Time) Third Semester)

Time : Three hours

Maximum : 100 marks

Use of approved statistical table permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Two dice are thrown together. Find the probability that (a) the total of the numbers on the top face is 9 and (b) the top faces are same.
2. A continuous random variable X that can assume any value between $x = 2$ and $x = 5$ has a density function given by $f(x) = k(1+x)$. Find $P(X < 4)$.
3. Define the Poisson and Normal distributions.
4. If X is uniformly distributed in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, find the pdf of $Y = \tan X$.
5. Find the marginal density functions of X and Y from the joint density function.

$$f(x, y) = \begin{cases} \frac{2}{5}(2x + 3y), & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

6. State central limit theorem.
7. Write the application of 'F'-test and χ^2 (chi-square test).
8. Define errors in sampling and critical region.

9. Write down the format of the ANOVA table for two factors of classification.

10. Write any two differences between RBD and LSD.

PART B — (5 × 16 = 80 marks)

11. (i) When a die is thrown, X denotes the number that turns up. Find $E(X)$, $E(X^2)$ and $\text{var}(X)$. (3)

(ii) A continuous random variable has a pdf $f(x) = kx^2e^{-x}$; $x \geq 0$. find k , mean and variance. (5)

(iii) Obtain the moment generating function of the random variable X having probability density functions $f(x) = \begin{cases} x, & 0 \leq x < 1 \\ 2 - x, & 1 \leq x < 2 \\ 0, & \text{otherwise.} \end{cases}$ (8)

12. (a) (i) State geometric distribution and find its mean and variance. (8)

(ii) In a component manufacturing industry, there is a small probability of $1/500$ for any component to be defective. The components are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing (A) no defective (B) two defective components in a consignment of 10,000 packets. (8)

Or

(b) (i) Define Exponential distribution and prove the memoryless property. (8)

(ii) The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = 1/2$. What is the probability that a repair time exceeds $2h$? What is the conditional probability that a repair time takes at least $10h$ given that its duration exceeds $9h$? (8)

13. (a) From the following data, find (16)

(i) the two regression equations.

(ii) the coefficient of correlation between the marks in economics and statistics.

(iii) the most likely marks in statistics when marks in economics are 30.

Marks in Economics	25	28	35	32	31	36	29	38	34	32
Marks in Statistics	43	46	49	41	36	32	31	30	33	39

Or

(b) Two random variables X and Y have joint pdf

$$f(x, y) = \begin{cases} \frac{xy}{96}, & 0 < x < 4, 1 < y < 5 \\ 0, & \text{otherwise} \end{cases}$$

Find $E(X)$, $E(Y)$, $E(XY)$, $E(2X + 3Y)$, $V(X)$, $V(Y)$, $Cov(X, Y)$. What can you infer from $Cov(X, Y)$? (16)

14. (a) Two random samples gave the following results.

Sample	Size	Sample mean	Sum of the squares of deviations from the mean
1	10	15	90
2	12	14	108

Examine whether the samples come from the same normal population. (16)

Or

(b) (i) Two researchers adopted different sampling techniques while investigating the same group of students to find the number of students falling into different intelligence level. The results are as follows: (8)

Research	Below average	Average	Above average	Excellent	Total
X	86	60	44	10	200
Y	40	33	25	2	100
Total	126	93	69	12	300

Would you say that the sampling techniques adopted by the two researchers are significantly different?

(ii) A machinist is making engine parts with axle diameters of 0.7 inch. A random sample of 10 parts shows a mean diameter of 0.742 inch with a standard deviation of 0.04 inch. Compute the statistic you would use to test whether the work is meeting the specification. (8)

15. (a) Analyze the variance in the latin square of yields (in kgs) paddy where P, Q, R, S denote the different methods of cultivation.

S122	P121	R123	Q122
Q124	R123	P122	S125
P120	Q119	S120	R121
R122	S123	Q121	P122

Examine whether the different methods of cultivation have given significantly different yields. (16)

Or

- (b) The following data represent the number of units of production per day turned out by different workers using 4 different types of machines:

		Machine type			
		A	B	C	D
Workers	1	44	38	47	36
	2	46	40	52	43
	3	34	36	44	32
	4	43	38	46	33
	5	38	42	49	39

Test whether the five men differ with respect to mean productivity and test whether the mean productivity is the same for the four different machine types.

(16)