A 376

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2005.

Fifth Semester

Chemical Engineering

MA 037 — SPECIAL FUNCTIONS, DIFFERENCE EQUATIONS AND Z–TRANSFORMS

(Common to Leather Tech. and Textile Tech.)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Evaluate $\int_0^1 (\log 1/y)^{n-1}\, dy$.

2. Compute $\Gamma(-5/2)$.

3. Find $P_0(x)$.

4. Evaluate $\int_{-1}^{+1} P(x)dx$.

5. State the recurrence relation connecting $J_n(x)$, $J_{n-1}(x)$ and $J_{n+1}(x)$.

6. Define Bessel Differential equation.

7. Show that $L_1(x) = 1 - x$.

8. Give the orthogonal property of Hermite polynomials.

9. Find $z(1/n)$.

10. Show that $z(a^n f(n)) = \bar{f}(z/a)$.

PART B — (5 × 16 = 80 marks)

11. (i) Prove that $P_n(x) = \frac{1}{n!2^n} D^n (x^n - 1)^n$. (8)

(ii) State and prove the orthogonal property of Legendre polynomials. (8)

12. (a) (i) Show that $P(m)P(m+1/2) = \frac{\sqrt{\pi}}{2^{2m-1}} P(2m)$. (8)

(ii) Evaluate $\int_0^1 x^m (1-x^n)^p dx$ in terms of Gamma function and hence find $\int_0^1 \frac{dx}{\sqrt{1-x^n}}$. (8)

Or

(b) Find the general solution in series of powers of 'x' of the equation.

$$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2ky = 0.$$ (16)

13. (a) (i) Show that $2n J_n(x) = x[J_{n-1}(x) + J_{n+1}(x)]$. (8)

(ii) Show that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$. (8)

Or

(b) (i) Prove that $\sin x = 2(J_1 - J_3 + J_5 \cdots)$. (8)

(ii) Show that $x J_n'(x) = n J_n(x) - x J_{n+1}(x)$. (8)

14. (a) (i) Show that $H_n'(x) = 2n H_{n-1}(x)$. (8)

(ii) Show that $H_{2n}(o) = (-1)^n \frac{2n!}{n!}$ and $H_{2n+1}(o) = 0$. (8)

Or

(b) (i) Show that $L_n(x) = \frac{e^x}{n!} D^n (x^n e^{-x})$. (8)

(ii) Show that $L_n'(x) = L_{n-1}'(x) - L_{n-1}(x)$. (8)

15. (a) (i) If $z[f(n)] = \bar{f}(z)$, show that $\underset{n\to\infty}{Lt} f(n) = \underset{z\to 1}{Lt} [\bar{f}(z)\cdot(z-1)]$. (8)

(ii) Find the z–transform of $\frac{1}{n(n+1)}$. (8)

Or

(b) (i) Find $z^{-1}\left[\frac{3z^2 - 18z + 36}{(z-2)(z-3)(z-4)}\right]$. (6)

(ii) Solve $y(n+3) - 3y(n+1) + 2y(n) = 0$ given that $y(0) = 4$, $y(1) = 0$ and $y(2) = 8$. (10)
