D. C.

E 9209

B.Sc. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2005.

Second Semester

Computer Technology

BCS 122 — ANALYTICAL GEOMETRY AND REAL AND COMPLEX ANALYSIS

(Common to B.Sc. Apparel and Fashion Technology)

(Regulations 2003)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Evaluate $\int_0^{\pi}\int_0^{\sin\theta} r\, dr\, d\theta$.

2. Evaluate $\int_0^3\int_0^2\int_0^1 (x+1)^2 yz\, dx\,dy\,dz$.

3. Obtain $\nabla\cdot\bar{r}$ with the usual notations.

4. What is Green's theorem in a plane?

5. Write the equation of a plane in normal form.

6. Write the equation of a line passing through the point (1, –2, 4) and parallel to the straight line having the direction ratios (2, –5, 7).

7. Write the harmonic property of a complex analytic function $W = f(z) = u + iv$.

8. Show that the complex function $W = e^z$ is analytic in the complex plane.

9. Write Cauchy's integral formula.

10. Give an example for an isolated singularity in complex plane.

PART B — (5 × 16 = 80 marks)

11. (i) If $W = f(z) = u + iv$ is a complex analytic function then prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. (8)

(ii) If $W = f(z)$ is a complex analytic function if $u - v = e^x(\sin y - \cos y)$ then find u and v. (8)

12. (a) (i) Evaluate $\int_0^{2\pi} \frac{d\theta}{a + b\cos\theta}$ $(a > b)$ using Contour integration in complex plane. (8)

(ii) Expand $f(z) = \frac{7z - 1}{(z^3 + 6z^2 + 11z + 6)}$ in terms of Laurent's series in the region $2 < |z| < 3$. (8)

Or

(b) (i) Evaluate $\int_0^{\infty} \frac{x^2 - 5}{(x^2 + 9)(x^2 + 16)} dx$ using contour integration in complex plane. (8)

(ii) If $\phi(\text{[illegible]}) = \int_C \frac{3z^2 + 7z + 1}{z - \text{[illegible]}} dz$, where C is the circle $x^2 + y^2 = 4$, find the values of (1) $\phi(3)$, (2) $\phi'(1-i)$ and (3) $\phi''(1-i)$. (8)

13. (a) (i) Obtain the gradient of the scalar function $f(x,y,z) = xy^2e^{2z} - 6$ at (1, –1, 3) in the direction of the vector $2\hat{i} - 7\hat{j} + 5\hat{k}$. (8)

(ii) Using Stoke's theorem in $x - y$ plane, evaluate $\oint_C \vec{F} \cdot d\vec{r}$ taken around the rectangle bounded by the lines $x \pm a$, $y = 0$, $y = b$. (8)

Or

(b) (i) With the usual notations, show that $\nabla^2(r^n) = n(n+1)r^{n-2}$. (8)

(ii) Evaluate $\iint(x\,dy\,dz + y\,dz\,dx + 2dx\,dy)$ over the surface of a sphere of radius 'a'. (8)

14. (a) (i) Show that the lines $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$ and $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$ are Coplanar. Find their common point and the equation of the plane in which they lie. (8)

(ii) Determine the equation of the sphere having the circle $x^2+y^2+z^2=16$, $x-y+z=3$ as a great circle. (8)

Or

(b) (i) Obtain the equation of the plane through the line $\frac{x-1}{3}=\frac{y-4}{2}=\frac{z-4}{-2}$ and parallel to the line $\frac{x+1}{2}=\frac{y-1}{-4}=\frac{(z+2)}{1}$. Hence find the shortest distance between them. (8)

(ii) Obtain the image of the point (–1, 3, 5) in the plane $3x-y+z=3$ and also the foot of the perpendicular from the above point to the plane given above. (8)

15. (a) (i) Evaluate $\iint \frac{r\,dr\,d\theta}{\sqrt{a^2+r^2}}$ over one loop of the lemniscate $r^2=a^2\cos 2\theta$. (8)

(ii) Obtain the volume of the ellipsoid.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1. \quad (8)$$

Or

(b) (i) Evaluate the following integral by changing to spherical coordinates $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx\,dy\,dz}{\sqrt{1-x^2-y^2-z^2}}$. (8)

(ii) Change the order of integration in $I=\int_0^1 \int_{x^2}^{2-x} xy\,dx\,dy$ and hence evaluate the same. (8)

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