B 476

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2005.

Fourth Semester

Metallurgical Engineering

MA 039 — PROBABILITY AND STATISTICS

(Common to Industrial Bio-Technology)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

Statistical tables are Permitted.

PART A — (10 × 2 = 20 marks)

1. Find the p.d.f. if exists

$$F(x) = \begin{cases} \frac{1}{2} e^{-\left(\frac{x}{50}\right)^2}, & x \le 0 \\ 1 - \frac{1}{2} e^{-\left(\frac{x}{50}\right)^2}, & x > 0. \end{cases}$$

2. Define Weibul distribution.

3. If (X, Y) is a continuous bivariate random variable with joint p.d.f. $f_{XY}\ (X, Y)$, define conditional probability density function of Y given X.

4. Find the mean of X and Y, given two regression lines are $x + 6y = 4$ and $2x + 3y = -1$.

5. Prove that a first order stationary random process has a constant mean.

6. State True or False, "the difference of two independent Poisson Process is a Poisson Process".

7. State the MTTF and the probability of a component which will fail in linearly increasing hazard model.

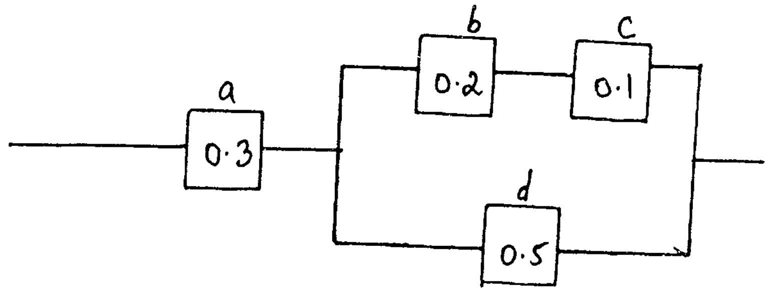
8. In a four element system, each having a reliability of 0.92, draw and calculate the reliability for series arrangement.

9. State the assumptions made in the analysis of variance.

10. Define 'attributes', give example.

PART B — (5 × 16 = 80 marks)

11. (i) Calculate the system reliability. (8)



(ii) A component in a system has a constant failure rate of 0.1 per day. Once it has failed, the mean time to repair is 2.5 days (1) what are the point availability at the end of 3 days and the steady state availability? (2) If one component operates in a stand by mode with no failure in standby, what is the steady-state availability? (8)

12. (a) (i) Find the mgf of normal distribution. (8)

(ii) If X is a random variable with pdf $f_X(x)$ and distribution function $F_X(x)$, obtain the distribution function and pdf of $Y = X^2$. (8)

Or

(b) (i) Prove that for a Poisson distribution (8)

$$\mu_{r+1} = r\lambda\ \mu_{r-1} + \lambda \frac{d\mu_r}{d\lambda}.$$

(ii) A fair die is tossed 720 times. Use Chebycheff's inequality to find a lower bound for the probability of getting 100 to 140 sixes. (8)

13. (a) (i) The joint probability density function of two random variables X and Y is given by $f(x, y) = \dfrac{9(1+x+y)}{2(1+x)^4(1+y)^4}$, $0 \le x \le \infty$, $0 \le y \le \infty$.

Find the marginal distribution of X and Y and the conditional distribution of Y for $X = x$. (8)

(ii) If a 1-gallon can of paint covers on the average 513.3 square feet with standard deviation 31.5 square feet, what is the probability that mean area covered by a sample of 40 of these 1-gallon cans will be anywhere from 510.0 to 520.0 square feet? (8)

Or

(b) (i) Two random variables X and Y have the following joint probability density function

$$f(x, y) = 2 - x - y, \; 0 \le x \le 1, 0 \le y \le 1$$
$$= 0, \qquad \text{otherwise.}$$

Find co-variance between X and Y. (4)

(ii) Prove that an arithmetic mean of the regression coefficients is greater than or equal to the correlation coefficient. (4)

(iii) If X and Y are two independent random variables with pdf's $f_x(X) = \alpha e^{-\alpha x}$, $x > 0, f_y(Y) = \beta e^{-\beta Y}$, $y > 0$; find pdf of $X - Y$. (8)

14. (a) (i) Is the process $\{X(t), t \in T\}$ with probability density

$$P(X(t) = n) = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n-1}}, & n = 1, 2, 3, \cdots \\ \dfrac{at}{1+at}, & n = 0 \end{cases}$$

stationary? (8)

(ii) The arrivals at a counter in a bank occur in accordance with a Poisson process at an average rate of 8 per hour. The duration of service of a customer has exponential distribution with a mean of 6 minutes. Find the probability that an arriving customer , (1) has to wait on arrival, (2) finds 4 customers in the system and (3) has to spend less than 15 minutes in the bank. Estimate also the fraction of the total time that the counter is busy. (8)

Or

(b) (i) The number of accidents in a city is modelled as a Poisson process with a mean of 2 per day. The number X_i of people involved in the ith accident has independent distribution $P(X_i = k) = \dfrac{1}{2K}(k \ge 1)$. Find the mean and variance of the number of people involved in accident per weak. (8)

(ii) A fair die is tossed repeatedly. If X_n denotes the maximum of the numbers occurring in the first n trials. find the transition probability matrix p of the Markov chain $\{X_n\}$. Also find p^2 and $P(X_2 = 6)$. (8)

15. (a) (i) Two pieces of cloth out of different rolls contained respectively 1, 4, 3, 2, 5, 4, 6, 7, 2, 3, 2, 5, 7, 6, 4, 5, 2, 1, 3 and 8 imperfections. Ascertain whether the process is in a state of statistical control. (8)

(ii) A machine is set to deliver packets of a given weight. Ten samples of size 5 each were recorded. Below are given data. Draw R-chart and comment on its state of control. (8)

Sample No. :	1	2	3	4	5	6	7	8	9	10
Range :	7	7	4	9	8	7	12	4	11	5

Or

(b) Five varieties of wheat *A, B, C, D* and *E* were tried. The gross size of the plot was 18 feet × 22 feet, the net plot being 14 feet × 18 feet × 22 feet, the net plot being 14 feet × 18 feet. Thus the whole experiment occupied an area 90 feet × 110 feet. The plan, the varieties shown in each plot and yields obtained in kg, are given in the following data :

B 90	E 80	C 134	A 112	D 92
E 85	D 84	B 70	C 141	A 82
C 110	A 90	D 87	B 84	E 69
A 81	C 125	E 85	D 76	B 72
D 82	B 60	A 94	E 85	C 88

Carry out an analysis of variance. What inference can you drawn from the data given? (16)