G 118

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2003.

Second Semester

Computer Science and Engineering

MA 035 — DISCRETE MATHEMATICS

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Using truth table verify that the proposition $(P \wedge Q) \wedge \neg(P \vee Q)$ is a contradiction.

2. Show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$ (use only the laws).

3. Show that $C(n,r) = C(n-1,\ r-1) + C(n-1,\ r)$.

4. Show that $2^n > n^3$, for $n \geq 10$, using induction principle.

5. Prove that monoid homomorphism preserves invertibility and monoid epimorphism preserves zero element (if it exists).

6. Prove that a subset $S (\neq \phi)$ of a group $(G, *)$ is a subgroup if and only if for any pair of elements $a, b \in S$, $a * b^{-1} \in S$.

7. Consider $(D_4, \leq)$ and $(D_9, \leq)$, where for positive integer n, D_n denotes the set of all positive divisors of n and $\leq$ is the divisibility. Obtain the Hasse diagram of the partially ordered set $L = D_4 \times D_9$ under the product partial order.

8. Prove that every distributive lattice is modular but not conversely.

9. Define graph isomorphism. Give an example for a pair of non–isomorphic graphs which have equal number of vertices, equal number of edges and the same degree sequence.

10. Prove that every non–trivial tree has at least two vertices of degree 1.

PART B — ($5 \times 16 = 80$ marks)

11. (i) Obtain the DNF and CNF for

$$(P \to (Q \wedge R)) \wedge (\neg P \to (\neg Q \wedge \neg R)).$$ (8)

(ii) Show that $(x)(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$ by using indirect method. (8)

12. (a) (i) Find the number of integers between 1 and 250 that are not divisible by any of the integers 2, 3, 5 and 7. (10)

(ii) Solve the recurrence relation

$$a_n = 2(a_{n-1} - a_{n-2}), \text{ where } n \geq 2 \text{ and } a_0 = 1,\ a_1 = 2.$$ (6)

Or

(b) (i) State and prove the Pigeonhole principle. (3)

(ii) For $m \in Z^+$ and m odd, prove that there exists a positive integer n such that m divides $2^n - 1$. (4)

(iii) Using generating function solve $a_n - 3a_{n-1} = n, n \geq 1,\ a_0 = 1$. (9)

13. (a) (i) Let $(G, *)$ be a finite cyclic group generated by an element $a \in G$. If G is of order n, prove that $a^n = e$ and $G = \{a, a^2, \cdots, a^n = e\}$ where n is the least positive integer for which $a^n = e$. (6)

(ii) Prove that every finite group of order n is isomorphic to a permutation group of degree n. (10)

Or

(b) (i) Prove that the order of a subgroup of a finite group divides the order of the group. (10)

(ii) Prove that the kernel of a homomorphism g from group $(G, *)$ to another group (H, Δ) is a normal subgroup of $(G, *)$. (4)

(iii) Define a field. (2)

14. (a) (i) Let $(L, *, \oplus)$ be an algebraic lattice. If we define $x \leq y \Leftrightarrow x * y = x$ or $x \leq y \Leftrightarrow x \oplus y = y$, then prove that $(L, \leq)$ is a lattice ordered set. (8)

(ii) Prove that the set of all positive integers ordered by divisibility is a distributive lattice. (8)

Or

(b) (i) Prove that the lattice of normal subgroups of a group C (with set inclusion) is a modular lattice. (6)

(ii) Prove that the elements of an arbitrary lattice satisfy distributive inequalities. (6)

(iii) Define Boolean algebra. Is there a Boolean algebra with five elements. Justify. (4)

15. (a) (i) Let G be a simple graph with minimum degree of at least two. Prove that there exists a cycle in G. (4)

(ii) Prove that the following statements are equivalent for a simple connected graph

(1) G is Eulerian

(2) Every vertex of G has even degree

(3) The set of edges of G can be partitioned into cycles. (12)

Or

(b) (i) When is a graph said to be self–complementary? Prove that if G is self–complementary then it has $4n$ or $4n+1$ vertices. (4)

(ii) Prove that a graph G is connected if and only if for any partition of V into subsets V_1 and V_2, there exists an edge joining a vertex of V_1 to a vertex of V_2. (6)

(iii) Show that K_n has a Hamiltonian cycle, for $n \geq 3$. What is the maximum number of edge–disjoint Hamilton cycles possible in K_n. Obtain all the edge–disjoint. Hamilton cycles in K_7. (6)
