G 119

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2003.

Second Semester

Information Technology

MA 039 — PROBABILITY AND STATISTICS

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If the probability that a communication system will have high fidelity is 0.81 and the probability that it will have high fidelity and high selectivity is 0.18, what is the probability that a system with high fidelity will also have high selectivity?

2. It is known that 5% of the books bound at a certain bindery have defective bindings. Find the probability that 2 of 100 books bound by this bindery will have defective bindings.

3. If two random variables have the joint density

$$f(x_1, x_2) = \begin{cases} x_1 x_2 & \text{for } 0 < x_1 < 1,\ 0 < x_2 < 2 \\ 0 & \text{elsewhere.} \end{cases}$$

Find the probability that both random variables will take on values less than 1.

4. If X is a normal random variable with mean zero and variance σ^2, find the probability density function of $y = e^X$.

5. If the customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between 2 consecutive arrivals is more than 1 minute.

6. If the transition probability matrix of a Markov chain is $\begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ find the steady–state distribution of the chain.

7. A system consisting of several identical components connected in parallel is to have a failure rate of atmost 4×10^{-4} per hour. What is the least number of components that must be used if each has a constant failure rate of 9×10^{-4}?

8. A system consists of 6 different components connected in series. Find the MTBF of the system, if the six components have exponential time to failure distributions with failure rate of 1.8, 2.4, 2.0, 1.3, 3.0 and 1.5 per 1000 hours respectively.

9. What is meant by completely randomized design?

10. Define a control chart.

PART B — ($5 \times 16 = 80$ marks)

11. (i) A random variable X has density function

$$f(x) = \frac{k}{1+x^2} \text{ if } -\infty < x < \infty$$

$$= 0 \quad \text{otherwise}$$

Determine K and the distribution function. Evaluate the probability $P(X \geq 0)$. (8)

(ii) The probability of a man hitting a target is 1/3. How many times must he fire so that the probability of hitting the target atleast once is more than 90%? (8)

12. (a) (i) If the joint probability density of two random variables is given by

$$f(x_1, x_2) = \begin{cases} 6e^{-2x_1 - 3x_2} & \text{for } x_1 > 0, x_2 > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Find the probability that the first random variable will take on a value between 1 and 2 and the second random variable will take on a value between 2 and 3. Also find the probability that the first random variable will take on a value less than 2 and the second random variable will take on a value greater than 2. (8)

(ii) If two random variables have the joint probability density

$$f(x_1, x_2) = \begin{cases} \frac{2}{3}(x_1 + 2x_2) & \text{for } 0 < x_1 < 1, 0 < x_2 < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the conditional density of the first given that the second takes on the value x_2. (8)

Or

(b) (i) If the joint density of X and Y is given by

$$f(x, y) = \begin{cases} (x + y)/3, & \text{for } 0 < x < 1, 0 < y < 2 \\ 0, & \text{otherwise} \end{cases}$$

Obtain the regression of Y on X and of X on Y. (8)

(ii) Let the random variable X have the marginal density $f_1(x) = 1$, $-\frac{1}{2} < x < \frac{1}{2}$ and let the conditional density of Y be

$$f(y|x) = 1, \ x < y < x + 1, \ -\frac{1}{2} < x < 0$$
$$= 1, \ -x < y < 1 - x, \ 0 < x < \frac{1}{2}$$

Prove that the variables X and Y are uncorrelated. (8)

13. (a) (i) Prove that the inter arrival time of a Poisson process, ie. the interval between two successive occurrences of a Poisson process with parameter λ, has an exponential distribution with mean $1/\lambda$. (8)

(ii) A machine goes out of order, whenever a component fails. The failure of this part follows a Poisson process with a mean rate of 1 per week. Find the probability that 2 weeks have elapsed since last failure. If there are 5 spare parts of this component in an inventory and that the next supply is not due in 10 weeks, find the probability that the machine will not be out of order in the next 10 weeks. (8)

Or

(b) (i) A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair dice and drove to work if and only if a 6 appeared. Find the probability that he takes a train on the third day and also the probability that he drives to work in the long run. (8)

(ii) A duplicating machine maintained for office use is operated by an office assistant who earns Rs. 5 per hour. The time to complete each job varies according to an exponential distribution with mean 6 minutes. Assume a Poisson input with an average arrival rate of 5 jobs per hour. If an 8–hour day is used as a base determine the percentage idle time of the machine and the average time a job is in the system. (8)

14. (a) (i) Suppose that the failure rate Z is given by

$$\begin{aligned} Z(t) &= 0,\ 0 < t < A \\ &= c,\ t \geq A \end{aligned}$$

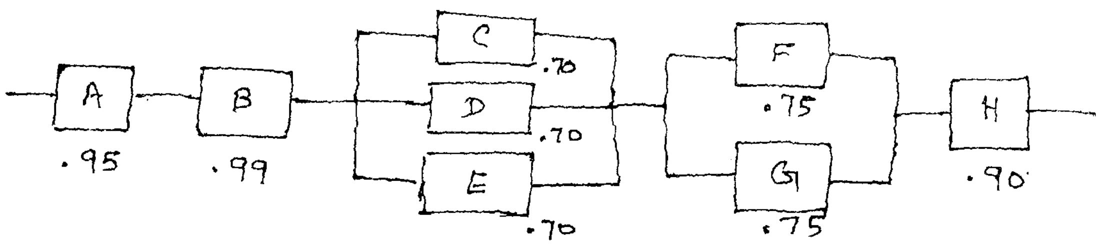
Find the probability density function associated with T, the time to failure. Evaluate $E(T)$. (8)

(ii) If a component has the Weibull failure time distribution with the parameters $\alpha = .005$ per hour and $\beta = .80$, find the probability that it will operate successfully for atleast 5000 hours. (8)

Or

(b) (i) A system consists of five independent components in series, each having a reliability of .970. What is the reliability of the system? What happens to the system reliability if its complexity is increased so that it contains 10 similar components? (8)

(ii) Find the reliability of the following system : (8)



15. (a) As part of the investigation of the collapse of the roof of a building, a testing laboratory is given all the available bolts that connected the steel structure at three different positions on the roof. The forces required to shear each of these bolts (coded values) are as follows :

Position 1 :	90	82	79	98	83	91	
Position 2 :	105	89	93	104	89	95	86
Position 3 :	83	89	80	94			

Perform an analysis of variance to test at the 0.05 level of significance whether the differences among the sample means at the three positions are significant.

Or

(b) Ten inspection sample lots of five amplifiers each are drawn from production. The following table lists the power output obtained for each amplifier. Plot an $\overline{X}$ and R chart and discuss whether the process is in control.

Sample No. :	1	2	3	4	5	6	7	8	9	10
Mean $\overline{X}$:	11	12	12.8	14	13.6	12.8	11.8	12.6	13.0	11.8
Range R :	4	4	6	4	6	5	5	6	4	6

Control factors for $n = 5$ are $A_2 = .58$, $D_3 = 0$, $D_4 = 2.115$.