G 218

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2003.

Fourth Semester

Computer Science Engineering

MA 040 — PROBABILITY AND QUEUEING THEORY

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. A is known to hit the target in 2 out of 5 shots whereas B is known to hit the target in 3 out of 4 shots. Find the probability of the target being hit when they both try?

2. The life time of a component measured in hours is Weibull distribution with parameter $\alpha = 0.2$, $\beta = 0.5$. Find the mean lifetime of the component.

3. If X is uniformly distributed in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, find the probability distribution function of $y = \tan x$.

4. The following table gives the joint probability distribution of X and Y. Find the
   (a) marginal density function of X
   (b) marginal density function of Y.

Y \ X	1	2	3
1	0.1	0.1	0.2
2	0.2	0.3	0.1

5. Find the mean of the stationary process $\{X(t)\}$ whose auto correlation function $R(z) = \dfrac{25z^2 + 36}{6.25z^2 + 4}$.

6. What is continuous random sequence? Give an example.

7. What is stochastic matrix? When is it said to be regular?

8. Define irreducible Markov chain? And state Chapman–Kolmogorov Theorem.

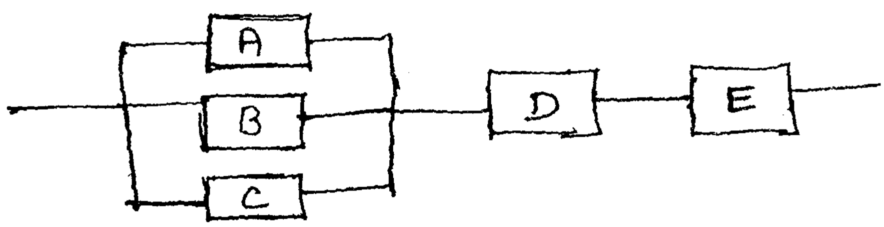
9. What is the probability that a customer has to wait more than 15 minutes to get his service completed in (M/M/1) : (∞/FIFO) queue system if $\lambda = 6$ per hour and $\mu = 10$ per hour?

10. Write down Pollaczek–Khintchine formula.

PART B — ($5 \times 16 = 80$ marks)

11. (i) Three boys A, B, C are throwing a ball to each other. A always throw the ball to B and B always throws the ball to C but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition matrix and classify the states. (8)

(ii) What is the reliability of the system shown in the following figure? $P(A) = P(B) = P(C) = 0.8$ (parallel redundancy) $P(D) = 0.95$, $P(E) = 0.85$. How would the reliability improve further if sub system E is also made parallel redundant? Show the configuration of this system. (8)



Figure

12. (a) (i) In a certain binary communication channel, owing to noise, the probability that a transmitted zero is received as a zero is 0.95 and the probability that a transmitted one is received as one is 0.9. If the probability that a zero is transmitted is 0.4, find the probability that

(1) a one is received

(2) a one was transmitted given that one was received. (8)

(ii) Find the M.G.F. and r^{th} moment for the distribution whose p.d.f. is $f(x) = ke^{-x}$ $0 \le x \le \infty$. Find also standard deviation. (8)

Or

(b) (i) 6 dice are thrown 729 times. How many times do you expect atleast three dice to show 5 or 6? (8)

(ii) A pair of dice be rolled 900 times and X denote the no. of times a total of 9 occurs. Find $P(80 \le x \le 120)$ using Chebyshev inequality. (8)

13. (a) (i) If the joint density function of the two random variables X and Y be

$$f(x, y) = e^{-(x+y)} \quad x \ge 0,\ y \ge 0$$
$$= 0 \qquad \text{otherwise.}$$

Find : (1) $P(x < 1)$ (2) $P(X + Y < 1)$. (8)

(ii) The joint p.d.f. of two variables X and Y is

$$f(x, y) = x + y \quad 0 < x < 1,\ 0 < y < 1$$
$$= 0 \qquad \text{otherwise.}$$

Find the correlation coefficient between x and y. (8)

Or

(b) (i) If X and Y are independent random variables each normally distributed with mean zero and variance σ^2 find the density function of $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$. (8)

(ii) Find the coefficient of correlation and obtain the lines of regression from the data given below : (8)

x :	62	64	65	69	70	71	72	74
y :	126	125	139	145	165	152	180	208

14. (a) (i) Prove that the difference of two independent poisson processes is not a poisson process. (6)

(ii) Show that the random process $X(t) = A\cos(wt + \theta)$ is wide sense stationary if A and w are constant and θ is uniformly distributed random variable in $(0, 2\pi)$. (10)

Or

(b) (i) The process $\{X(t)\}$ whose probability distribution under certain condition is given by

$$P\{X(t) = n\} = \frac{(at)^{n-1}}{(1+at)^{n+1}} \quad n = 1, 2, \ldots.$$

$$= \frac{at}{1+at} \quad n = 0$$

Show that it is not stationary. (8)

(ii) Given that WSS random process $X(t) = 10\cos(100t + \theta)$ where θ is uniformly distributed over $(-\pi, \pi)$. Prove that the process $X(t)$ is correlation – ergodic. (8)

15. (a) (i) A repairman is to be hired to repair machines which breakdown at an average rate of 3 per hour. The breakdown follow poisson distribution. Non–productive time of machine is considered to cost Rs. 16/hour. Two repairman have been interviewed. One is slow but cheap while the other is fast and expensive. The slow repairman charges Rs. 8 per hour and he services machines at the rate of 4 per hour. The fast repairman demands Rs. 10 per hour and services at the average rate of 6 per hour. Which repairman should be hired? (8)

(ii) A two person barber shop has 5 chairs to accommodate waiting customers. Potential customers who arrive when all 5 chairs are full, leaving without entering barber shop. Customers arrive at the average rate of 4 per hour and spend an average of 12 minutes in the barber's chair. Compute ρ_0, ρ_7 and average number of customers in the queue. (8)

Or

(b) (i) Derive the formula for :

(1) average number L_q of customers in the queue

(2) average waiting time of a customer in the queue for (M/M/1) : (∞/FIFO) model. (8)

(ii) On average 96 patients per 24 hour day require the service of an emergency clinic. Also on average a patient requires 10 minutes of active attention. Assume that the facility can handle only one emergency at a time. Suppose that it costs the clinic Rs. 100 per patient treated to obtain an average servicing time of 10 minutes and that each minute of decrease in this average time would cost Rs. 10 per patient treated. How much would have to be budgeted by the clinic to decrease the average size of the queue from $1\frac{1}{3}$ patient to $\frac{1}{2}$ patient? (8)

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