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M.C.A. DEGREE EXAMINATIONS: NOVEMBER 2009

Second Semester

P07CA201 Mathematical Foundations of Computer Science

Time: Three Hours

Maximum Marks: 100

Answer ALL the Questions:-

PART A (10 x 2 = 20 Marks)

1) Find the eigen values and eigen vectors of the matrix $A=\begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix}$

2) Prove that the eigen values of $(-3A^{-1})$ are the same as those of $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

3) If $f : Z \rightarrow N$ is defined by

$$f(x) = \begin{cases} 2x-1 & \text{if } x > 0, \\ -2x & \text{if } x \leq 0, \end{cases}$$

Is f a bijective function ?

4) Let A = {a, b, c, d } and R be the relation defined on A whose matrix representation is

$$M_R = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Is R symmetric? , Is R reflexive?

5) Symbolize the following expression "X is the father of the mother of Y"

6) Find the PDNF of $(Q \rightarrow P) \wedge (\neg P \wedge Q)$

7) Find the language L (G) of the grammar G ={(S,A), (a, b), S, P)} where
P = { S→ a A, S→ b, A→ aa}

8) Determine the type of the grammar G which consist of the following
Products: S→ a B, B→ b A, B→ b, B→ a, A→ a B, A → a.

9) Define finite state automata and give an example.

10) How will you convert an NFA into an equivalent DFA.

PART B (5 x 16 =80 Marks)

11 a) i) Verify Cayley – Hamilton theorem for the matrix

$$A = \begin{pmatrix} 7 & 2 & 2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix}$$ and hence find A^{-1} (8)

ii) Investigate for what values of λ and μ the equations $x + y + 2z = 2, 2x - y + 3z = 2$ and $5x - y + \lambda z = \mu$ have i) no solution ii) a unique solution iii) an infinite number of solution (8)

(OR)

b) i) Prove that the eigen values of a real symmetric matrix are real. (6)

ii) Find the eigen values and eigen vectors of (adj A), given that the matrix (10)

$$A = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{pmatrix}$$

12 a) i) In a class of 25 students, 12 have taken mathematics, 8 have taken mathematics but not biology. Find the number of students who have taken mathematics and biology and those who have taken biology but not mathematics. (8)

ii) If A, B, C are any three sets, then prove that $A - (B \cup C) = (A-B) \cap (A-C)$ (8)

(OR)

b) i) (a, b) R (c, d) if and only if $a + 2b = c + 2d$, where a, b, c and d are real, prove that R is an equivalence relation. (8)

ii) f: $N \times N \rightarrow N$ given by $f(m, n) = 14m + 22n$, Is f bijective ? (8)

13 a) i) Obtain principal conjunctive normal form of $A = (p \wedge q) \vee (\neg p \wedge q) \vee (q \wedge r)$ by constructing principal disjunctive normal form. (8)

ii) Check the validity of the following argument "If Roli has completed MCA or MBA, then she is assured of a good job. If Roli is assured of a good job, she is happy. Roli is not happy. So Roli has not completed MBA" (8)

(OR)

b) i) Prove that $p \vee q, p \rightarrow r, q \rightarrow s \implies s \vee r$ by direct method. (6)

ii) Show that the premises "One student in this class knows how to write programs in JAVA" and "Everyone who knows how to write programs in JAVA can get a high-paying job" imply the conclusion "Someone in this class can get a high-paying job". (10)

14 a) i) Find the phase-structure grammar that generates the set $L = \{0^n 1^n : n \geq 0\}$ (8)

ii) Construct derivation trees for the words (i) ababbbba, (ii) bbbcbba using the grammars G1 and G2 respectively, where G1 consists of the productions (8)
$\{S \rightarrow AbS,\ A \rightarrow aS,\ S \rightarrow ba$ and $A \rightarrow b\}$ and G2 consists of the productions
$\{S \rightarrow bcS,\ S \rightarrow bbS,\ S \rightarrow cb,\ S \rightarrow a\}$.

(OR)

b) i) Find a grammar that generates the set of words $\{a^n b^n c^n \ / \ n \geq 1\}$ (8)

ii) Show that the grammar $G = \{(S,A), (a, b), S, P)\}$ where $P = \{S \rightarrow aS,\ S \rightarrow aSb,$
$S \rightarrow ab\}$ is ambiguous. (8)

15 a) i) Find the DFA equivalent to the NFA for which the state- table is given in following Table and s_2 is the accepting state.

I	*f*	
S	a	b
s_0	s_0, s_1	s_2
s_1	s_0	s_1
s_2	s_1	s_0, s_1

(OR)

b) i) Design an Finite State Machine(FSM) that performs serial binary addition. (10)

ii) Design an Finite State Machine (FSM) that outputs 1, if an even number of 1's have been input and outputs 0 otherwise. (6)

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