Register Number: ……………………..

B.E / B.TECH DEGREE EXAMINATIONS: MAY/JUNE 2014

(Regulation 2009)

First Semester

Common to all Branches

MAT101: Engineering Mathematics I

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. If the Eigen values of the matrix A are 1 and 5, then the eigen values of the matrix $3A^2$ are

   a) 3 & 25 b) 9&75
   c) 1&25 d) 3&75

2. The nature of the quadratic form $f(x_1, x_2, x_3) = x_1 + 2x_2^2$ is

   a) positive definite b) indefinite
   c) positive semi definite d) negative semi definite

3. The equation of the plane passing through the point (1, 2, 3) and which is parallel to the plane $4x+5y-3z+7=0$ is

   a) $4x+5y-3z=0$ b) $4x+5y-3z+6=0$
   c) $4x+5y-3z+7=0$ d) $4x+5y-3z-5=0$

4. If two spheres $x^2+y^2+z^2+6y+2z+8=0$ and $x^2+y^2+z^2+6x+8y+az+20=0$ intersect at right angle, then the value of a is

   a) 4 b) 20
   c) 6 d) 8

5. The reciprocal of the curvature is called

   a) radius of curvature b) evolute
   c) centre of curvature d) circle of curvature

6. The envelope of the family of straight lines $y = mx+\frac{3}{2m}$ is

   a) $y^2 = 4x$ b) $x^2 = 6y$
   c) $x^2 = 4y$ d) $y^2 = 6x$

7. If $f(x,y)=e^x \log(1+y)$ then the value of $\frac{\partial f}{\partial x}$ at the origin is

   a) 1 b) $\log(1+y)$
   c) -1 d) 0

8. If $y = f(x)$ then the condition for local maximum is

   a) $\frac{df}{dx}=0 \,\&\, \frac{d^2f}{dx^2}>0$ b) $\frac{df}{dx}=0 \,\&\, \frac{d^2f}{dx^2}=0$
   c) $\frac{df}{dx}>0 \,\&\, \frac{d^2f}{dx^2}>0$ d) $\frac{df}{dx}=0 \,\&\, \frac{d^2f}{dx^2}<0$

9. The particular integral of $(D^2-3D)y=2$ is

   a) $\frac{-2x}{3}$ b) 0
   c) 2x d) x^2

10. The solution of the differential equation $(xD^2+D)y=0$ is

    a) $y = A\cos x + B$ b) $y = A\sin 2x + B\cos 2x$
    c) $y = A\log x + B$ d) $y = A\sin x + B$

PART B (10 x 2 = 20 Marks)

11. Find the eigen values of the matrix $\begin{pmatrix} 6 & 10 \\ 14 & 25 \end{pmatrix}$.
12. Using Cayley-Hamilton theorem find A^{-1} *if* $A=\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$.
13. Find the acute angle in degrees between the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{-1}$ *and* $\frac{x}{2}=\frac{y}{1}=\frac{z}{1}$.
14. Find the equation of a sphere having the points (- 4, 5, 1) and (4, 1, 7) as ends of a diameter.
15. Find the curvature at (3, - 4) to the curve $x^2+y^2=25$.
16. Find the envelope of $(x-a)^2+(y-a)^2=2a$.
17. If $u = f(x-y, y-z, z-x)$ find $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}$.
18. Find the stationary points of $x^2+y^2+6x+12$.
19. Solve $(D^2-3D+2)y=\cos x$.
20. Reduce $(x+3)^2 y''-4(x+3)y'+6y=0$ into differential equation with constant co-efficient.

PART C (5 x 14 = 70 Marks)

21. a) (i) Find the eigen values and eigen vectors of the matrix: (7)

$$A=\begin{pmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{pmatrix}$$

(ii) Verify Cayley Hamilton theorem for the matrix and hence find A^{-1} (7)

$$A=\begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$$

(OR)

b) Reduce the quadratic form $3x^2+5y^2+3z^2-2xy-2yz+2xz$ to canonical form by orthogonal reduction. Also find the rank, index, signature and nature of the quadratic form.

22. a) (i) Show that the lines $\frac{x+4}{3}=\frac{y+6}{5}=\frac{z-1}{-2}$ and $3x-2y+z+5=0=2x+3y+4z-4$ are coplanar. Find their point of intersection and the plane in which they lie. (7)

(ii) Show that the sphere $2x-2y+z=9$ touches the sphere $x^2+y^2+z^2+2x+2y-7=0$ and find the point of contact (7)

(OR)

b) Find the length and equations of the shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ & $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$

23. a) (i) Find the circle of curvature of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at the point $\left(\frac{a}{4},\frac{a}{4}\right)$. (7)

(ii) Find the envelope of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ where a and b are connected by the relation $a^2-b^2=c^2$. (7)

(OR)

b) (i) Find the evolute of the parabola $y^2=4ax$. (7)

(ii) Show that the evolute of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $(ax)^{2/3}+(by)^{2/3}=\left(a^2+b^2\right)^{2/3}$. (7)

24. a) (i) If $u=xy+yz+zx, x=\frac{1}{t}, y=e^t, z=e^{-t}$ find $\frac{du}{dt}$. (6)

(ii) A rectangular box open at the top is to have a volume of 32cc. Find the dimensions of the box, that requires the least material for its construction. (8)

(OR)

b) (i) Find the maxima and minima for $f(x,y)=x^3+3xy^2-3x^2-3y^2+4$. (7)

(ii) Expand $e^x \sin y$ in powers of x and y up to the 2nd degree term. (7)

25. a) (i) Solve: $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=e^{-x}\sin 2x$. (7)

(ii) Solve the equation by method of variation of parameter $\frac{d^2y}{dx^2}+4y=\sec 2x$. (7)

(OR)

b) (i) Solve: $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=2\sin\log(1+x)$. (7)

(ii) Solve: $(D+4)x+3y=t; 2x+(D+5)y=e^{2t}$. (7)

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