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B.E DEGREE EXAMINATIONS:MAY/JUNE 2014

(Regulation 2009)

Second Semester

MAT102: ENGINEERING MATHEMATICS II

(Common to AERO, AUTO, CE, ECE, EEE, EIE, MECT & MECH)

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. The value of $\int_0^1\int_0^2\int_0^3 xyz\,dx\,dy\,dz$ is

   a) $\frac{9}{2}$ b) $\frac{9}{4}$

   c) $\frac{5}{2}$ d) $\frac{9}{4}$

2. Region of integration in the integral $\int_0^3\int_0^2 (4-y^2)\,dy\,dx$ is a

   a) rectangle b) square

   c) Parabola d) ellipse

3. If $\vec{r}$ is the position vector of a point in the xyz plane, then $div\vec{r}$ is equal to

   a) 1 b) 2

   c) 3 d) 0

4. The maximum value of the directional derivative of $\varphi(xyz) = x^2yz$ $at(1,4,1)$ *is*

   a) 19 b) 0

   c) -9 d) 9

5. The necessary condition for a function to be analytic is

   a) $u_x = v_y$ *and* $u_{xx} = v_{xy}$ b) $u_y = v_y$ *and* $u_x = v_x$

   c) $u_{xx} = v_{yy}$ *and* $u = v$ d) $u_x = v_y$ *and* $u_y = -v_x$

6. The invariant points of the transformation $w = \frac{2z+6}{z+7}$ are

   a) 1 *and* 6 b) -1 and -6

   c) 1 and -6 d) -1 and 6

7. If the Laurent's series of f(z) contains an infinite number of negative powers of (z-a) then z = a is called as

   a) Removable singularity b) Poles

   c) an isolated singularity d) an essential singularity

8. The value of $\int_c \frac{z}{z^2-1}dz$ where c is the circle |z| = ½ is

   a) 0 b) 6πi

   c) 18πi d) 14πi

9. The Laplace transform of e^{-2t}

   a) $\frac{1}{s-2}$ b) $\frac{1}{s+2}$

   c) $\frac{s}{s-2}$ d) $\frac{s}{s+2}$

10. The periodicity of $\sin(3x+7)$ is

    a) $\frac{3\pi}{2}$ b) $\frac{7\pi}{2}$

    c) $\frac{2\pi}{3}$ d) $\frac{2\pi}{7}$

PART B (10 x 2 = 20 Marks)

11. Transform the integral $\int_0^\infty\int_0^y dxdy$ into polar coordinates.

12. Evaluate $\int_0^1\int_0^2\int_0^3 xyzdxdydz$

13. In what direction from the point (2,1,-1) is the directional derivative of $f= x^2yz^3$ a maximum. What is its magnitude?

14. Find 'a' so that the vector $\vec{F} = (ax+3y)\vec{i} + (2y-3z)\vec{j} + (x-3z)\vec{k}$ is solenoidal.

15. If f(z) is analytic then show that f(z) is a constant if real part of f(z) = 0 is a constant.

16. When is the transformation said to be conformal?

17. Evaluate $\int_c \frac{e^z}{z-1}dz$ where c is the circle $|z| = 2$

18. Find the residue of the function $f(z)=\frac{z}{z^2+1}$ about z = i.

19. Define Laplace transformation.

20. Find $L^{-1}\left[\frac{1}{(s-3)^2}\right]$.

PART C (5 x 14 = 70 Marks)

21. a) (i) Change the order of integration and then evaluate $\int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} dydx$

(ii) Evaluate $\iiint_V dxdydz$ where V is the region of space inside the tetrahedron bounded by the planes x = 0, y = 0, z = 0 and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

(OR)

b) (i) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$

(ii) Find the area of the region outside the inner circle r = 2 cosθ and inside the outer circle r = 4 cosθ by double integration.

22. a) (i) Find the directional derivative of $\Phi = 2xy + z^2$ at the point (1,-1,3) in the direction $\vec{i}+2\vec{j}+2\vec{k}$.

(ii) Use Green's theorem in the plane, evaluate $\int_C (2x-y)dx+(x+y)dy$ where C is the boundary of the circle $x^2+y^2=1$ in the XOY plane.

(OR)

b) (i) Evaluate $\overrightarrow{F}\bullet\vec{n}ds$ *if* $\vec{F}=4xz\vec{i}-y^2\vec{j}+yz\vec{k}$ taken over the cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0,z = 1.

(ii) Evaluate $\int_C xydx+xy^2dy$ by Stoke's theorem where C is the square in the XY plane with vertices (1,0), (-1,0), (0,1) and (0,-1).

23. a) (i) If f(z) is an analytic function of z, Prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log|f(z)|=0$.

(ii) Find the image of the circle |z|=2 by the transformation w = z + 3 +2i.

(OR)

b) (i) Find the bilinear transformation that maps the points (∞,i,0) in the z-plane onto the points (0,i, ∞) in the w-plane.

(ii) Construct the analytic function $f(z)=u+iv$ given that $v=e^{2x}(x\cos 2y-y\sin 2y)$.

24. a) (i) Evaluate $\int_C \frac{e^z}{(z^2+\pi^2)^2}dz$ where C is |z| = 4 using Cauchy's residue theorem.

(ii) Expand $\frac{1}{(z-1)(z-2)}$ n Laurent's series valid for 0 < |z-1| < 1.

(OR)

b) (i) Evaluate $\int_0^\infty \frac{x\sin x}{x^2+a^2}dx$ using contour integration.

(ii) Obtain the Taylor's series expansion of $f(t)=\frac{1}{t(1-t)}$ about t = - 1.

25. a) (i) Find the Laplace transform of the periodic function

$$f(t)=\begin{cases} t & 0<t<a \\ -t+2a & a<t<2a \end{cases}$$

where f(t+2a) = f(t)

(ii) Using Convolution theorem, find $L^{-1}\left[\frac{1}{s(s^2+a^2)}\right]$

(OR)

b) Using Laplace transform, Solve $y''(t)-2y'(t)-8y(t)=0,\quad y(0)=3,\quad y'(0)=6$

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