Register Number: ……………………..

B.E/ B.TECH DEGREE EXAMINATIONS: MAY/JUNE 2014

(Regulation 2009)

Second Semester

MAT103: ENGINEERING MATHEMATICS II

(Common to CSE, FT, BIO, IT & TXT)

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. The value of integrals $\int_0^1\int_x^1 dydx$ is
   a) 2 b) ½
   c) 1 d) 0
2. The curve $r^2 = a^2\cos 2\theta$ is symmetrical about the line
   a) $\theta = 0$ b) r = a
   c) r = -a d) $\theta = \pi/2$
3. If $\varphi = x^2 + y^2 + z^2 - 8$ then $grad\,\varphi$ at (2,0,2) is
   a) $4\vec{i} + 4\vec{k}$ b) $\vec{i} + \vec{k}$
   c) $4\vec{j} + 4\vec{k}$ d) $\vec{i} + \vec{j}$
4. $\vec{F}$ is said to be irrotational if
   a) $\nabla.\vec{F} = 0$ b) $\nabla \times \vec{F} = 0$
   c) $\nabla\vec{F} = \varphi$ d) $\vec{F} = 0$
5. An analytic function with constant real part is
   a) 1 b) 0
   c) constant d) harmonic
6. The function f(z) = z is
   a) nowhere analytic b) differentiable
   c) not differentiable d) analytic at (0,0)
7. The transformation w = c z where 'c' is complex constant consists of
   a) translation b) magnification
   c) magnification and rotation d) Rotation

8. The critical points of $w^2 = (z-\alpha)(z-\beta)$ are
   a) α, β b) $\alpha, \beta, \alpha\beta$
   c) $\alpha, \beta, \frac{\alpha+\beta}{2}$ d) $\alpha, \alpha+\beta$
9. If there is no other singularity in the neighbourhood of 'a' then the point z = a is called
   a) essential singularity b) removable singularity
   c) an isolated singularity d) pole
10. The value of $\int_C \frac{dz}{z-2}$ where C is $|z| = 1$ is
   a) 2 b) 3
   c) 0 d) 1

PART B (10 x 2 = 20 Marks)

11. Find the value of $\int_0^{\pi/2}\int_0^2 r\,drd\theta$.
12. Evaluate $\int_0^4\int_1^2\int_0^3 x^2yz\,dz\,dx\,dy$.
13. Find the unit normal to the surface $xy^3z^2 = 4$ at the point (-1,-1,2).
14. State Gauss divergence theorem.
15. Define regular function.
16. If $f(z) = u + iv$ is an analytic function, then prove that $u - iv$ is not an analytic function.
17. Prove that the function $x^4 - 6x^2y^2 + y^4$ is harmonic.
18. Find the invariant points of $f(z) = \frac{1}{z-2i}$.
19. Find the value of $\int_c \frac{e^{-z}}{z^2}dz$, where C is $|z|=1$.
20. Expand e^{2z} about z = 2i as a Taylor's series.

PART C (5 x 14 = 70 Marks)

21. a) (i) Change the order of integration in $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dydx$ and hence evaluate. (7)

(ii) Find the smaller area bounded by $y=2-x$ and $x^2+y^2=4$. (7)

(OR)

b) (i) By changing to polar coordinates, evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\, dydx$. (7)

(ii) Express the volume of the sphere $x^2+y^2+z^2=a^2$ as a volume integral and hence evaluate it. (7)

22. a) (i) If $\nabla\varphi = 2xyz^3\vec{i} + x^2z^3\vec{j} + 3x^2yz^2\vec{k}$ find $\varphi(x,y,z)$ such that $\varphi(1,-2,2)=4$. (8)

(ii) Prove that $\nabla.(r^n\vec{r}) = (n+3)r^n$. (6)

(OR)

b) Verify Stoke's theorem for $\vec{F} = xy\,\vec{i} - 2yz\,\vec{j} - zx\,\vec{k}$ where S is an open surface of the rectangular parallelopiped formed by the planes x = 0,x = 1,y = 0, y = 2 and z = 3 above the XOY plane.

23. a) (i) Prove that $f(z)=\cosh z$ is an analytic function and find its derivative. (7)

(ii) If $f(z)$ is an analytic function, then prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. (7)

(OR)

b) Determine the analytic function $f(z)=u+iv$, given that

$u-v=\frac{\cos x+\sin x-e^{-y}}{2\cos x-e^{y}-e^{-y}}$ and $f(\frac{\pi}{2})=0$.

24. a) (i) Prove that the transformation $w=\frac{1}{z}$ transforms, in general, circles and straight lines into circles and straight lines. (7)

(ii) Find the image of the circle $|z|=2$ under the transformation $w=(1+2i)z+(3+4i)$. (7)

(OR)

b) (i) Discuss the transformation $w=e^z$. (7)

(ii) Find the bilinear transformation which maps the points $z=0,-1,\infty$ into the points $w=-1,-2,-i$ respectively. (7)

25. a) (i) Use Cauchy's integral formula to evaluate $\int_c \frac{\sin\pi z^2+\cos\pi z^2}{(z-2)(z-3)}dz$, where C is the circle $|z|=4$. (8)

(ii) Find the Laurent's series of $f(z)=\frac{1}{z(1-z)}$ valid in the region $1<|z+1|<2$. (6)

(OR)

b) Evaluate $\int_{-\infty}^{\infty}\frac{x^2}{(x^2+a^2)^2(x^2+b^2)}dx$ using Contour integration where a > b >0.

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