Register Number: ……………………..

B.E / B.TECH DEGREE EXAMINATIONS: APRIL/MAY 2014

(Regulations 2009)

Third Semester

MAT105: TRANSFORM METHODS IN ENGINEERING

(Common to CSE & IT)

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. A "periodic function" is given by a function which

   a) has a period $T=2\pi$ b) satisfies $f(t+T)=f(t)$

   c) satisfies $f(t+T)= -f(t)$ d) has a period $T=\pi$

2. The value to which the Fourier series $f(x)=\begin{cases}\pi+x, -\pi\le x\le 0\\ 0, \quad 0\le x<\pi\end{cases}$ converges at x = 0 is

   a) π b) $\pi/2$

   c) 2π d) 0

3. $L\left\{f(\frac{t}{a})\right\}=$________

   a) $a\,\phi(as)$ b) $\frac{1}{a}\phi(as)$

   c) $a\,\phi(s)$ d) $\frac{1}{a}\phi(s)$

4. $L\left[t^n f(t)\right]=$

   a) $(-1)^n\frac{d^n}{ds^n}\phi(s)$ b) $\frac{d^n}{ds^n}\phi(s)$

   c) $\frac{d}{ds}\phi(s)$ d) $(-1)^n\frac{d}{ds}\phi(s)$

5. $L^{-1}\left[\frac{1}{s+a}\right]$ is valid for

   a) $s>-a$ b) $s=a$

   c) $s>a$ d) $s=-a$

6. $L^{-1}\left[\phi(s+a)\right]$ is

   a) $e^{at}L^{-1}\left[\phi(s)\right]$ b) $e^{-at}L^{-1}\left[\phi(s/a)\right]$

   c) $e^{-at}L^{-1}\left[\phi(as)\right]$ d) $e^{-at}L^{-1}\left[\phi(s)\right]$

7. If $F\left(f(x)\right)=F(s)$, then $F\left(f(2x)\right)=$

   a) $\frac{1}{2}F\left(\frac{s}{2}\right)$ b) $2F(s)$

   c) $\frac{1}{2}F(s)$ d) $2F\left(\frac{s}{2}\right)$

8. $F\left[e^{-|x|}\right]=$

   a) does not exist b) $\sqrt{\frac{2}{\pi}}\left[\frac{2}{1+x^2}\right]$

   c) $\sqrt{\frac{2}{\pi}}.\frac{1}{1+s^2}$ d) $\sqrt{\frac{2}{\pi}}.\frac{s}{1+s^2}$

9. The Z – Transform of zero is

   a) Not defined b) 0

   c) 1 d) ∞

10. Z – transform of unit impulse sequence is

    a) 0 b) -1

    c) 1 d) ± 1

PART B (10 x 2 = 20 Marks)

11. Does f(x) = tan x possess a Fourier expansion? Justify.
12. If $\cos^3 t=\frac{a_o}{2}+\sum_{n=1}^{\infty}\left(a_n\cos nt+b_n\sin nt\right)$ in $0\le t\le 2\pi$, find the sum of the series $\frac{a_0^2}{4}+\sum_{1}^{\infty}\left(a_n^2+b_n^2\right)$.
13. Evaluate $\int_0^\infty t^2e^{-2t}dt$ using Laplace transform.
14. If $f(t)=e^{-2t}\sin 2t$, find $L\left[f'(t)\right]$.
15. Find the inverse Laplace transform of $\tan^{-1}s$.
16. Find $L^{-1}\left[\frac{1}{s^4-1}\right]$.
17. If $F(s)=F\left(f(x)\right)$, then prove that $F\left(x f(x)\right)=(-i)\frac{d\left[F(s)\right]}{ds}$.
18. State convolution theorem in Fourier Transform.

19. If $z\left[f(n)\right]=F(z)$ show that $z\left[f(-n)\right]=F\left(\frac{1}{z}\right)$

20. Find the Z transform of $2^n * 3^n$.

PART C (5 x 14 = 70 Marks)

21. a) (i) Find the Fourier series of period 2π for the function $f(x)=\sqrt{(1-\cos x)}$ in $-\pi < x < \pi$. (7)

(ii) The function f(x) is given by (7)

$$f(x) = \frac{1}{4}-x \ , \quad for \quad 0<x<\frac{1}{2}$$
$$= x-\frac{3}{4} \ , \quad for \quad \frac{1}{2}<x<1$$

Find its half range cosine series.

(OR)

b) (i) Find the Fourier series for the function (7)

$$f(x)=\begin{cases}-K\ , & -\pi<x<0\\ K\ , & 0<x<\pi\end{cases} \text{ and } \quad f(x+2\pi)=f(x)$$

(ii) The turning moment T on the crank-shaft of a steam engine for the crank angle θ is as follows. (7)

θ	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
T	0	2.7	5.2	7	8.1	8.3	7.9	6.8	5.5	4.1	2.6	1.2	0

Expand T in a series of sines upto second harmonics.

22. a) (i) Find the Laplace transform of i) $t^n e^{at}$ ii) $t^2 \sin 2t$ (7)

(ii) Find the Laplace transform of Unit step function and Dirac delta function. (7)

(OR)

b) (i) Find the Laplace transform of $f(t)=t^2e^{-3t}$ and verify initial value theorem. (7)

(ii) Find the Laplace Transform of $f(t)=\begin{cases}t, 0<t<1\\ 2-t, 1<t<2\end{cases}$ such that $f(t+2)=f(t)$ (7)

23. a) (i) Find the inverse Laplace transform of i) $\frac{3S+5\sqrt{2}}{S^2+8}$ ii) $\frac{S e^{-\frac{s}{2}}+\pi e^{-s}}{S^2+\pi^2}$ (6)

(ii) Solve the initial value problem $y''+2y'+y=e^{-t}, y(0)=-1, y'(0)=1.$ (8)

(OR)

b) (i) Use convolution theorem to find the inverse Laplace transform of $\frac{s}{(s^2+\pi^2)^2}$ (7)

(ii) Solve the integral equation $y(t)=t+\int_0^t y(\tau)\sin(t-\tau)d\tau$ (7)

24. a) Find the Fourier cosine transform of e^{-ax} and use it to find the Fourier transform of $e^{-a|x|}\cos bx$.

(OR)

b) Show that Fourier transform of $f(x)=\begin{cases}a^2-x^2, |x|\le a\\ 0, |x|>a>0\end{cases}$ is $2\sqrt{\frac{2}{\pi}}\left(\frac{\sin as - as\cos as}{s^3}\right)$.

Hence deduce that $\int_0^\infty \frac{\sin t - t\cos t}{t^3}dt=\frac{\pi}{4}$. Using Parseval's identity show that $\int_0^\infty \left(\frac{\sin t - t\cos t}{t^3}\right)^2 dt=\frac{\pi}{15}$.

25. a) (i) Find the Z – Transform of i) $a^n \sin n\theta$ ii) $\frac{1}{n(n-1)}$ (7)

(ii) Use convolution theorem to find the inverse Z – Transform of $\frac{1}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}$ (7)

(OR)

b) Solve the equation $y(n+2)+y(n)=2^n$ using Z – Transform. Given y(0)=1, y(1)=0.

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