Register Number: ……………………..

B.E / B.TECH DEGREE EXAMINATIONS: APRIL/MAY 2014

(Regulation 2009)

Fifth Semester

MAT108 : NUMERICAL METHODS

(Common to ECE/IT)

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. In Newton's method the error at any stage is _______________ to the square of the error in the previous stage
   a) reciprocal b) directly proportional
   c) equal d) inversely proportional
2. The equation $x^2+ax+b=0$ has two real roots α and β. Then in the iteration method $x_{k+1}=-(x_k^2+b)/a$ is convergent near α if
   a) $|\alpha|>|\beta|$ b) $|\alpha|<|\beta|$
   c) $2|\alpha|>|\alpha+\beta|$ d) $2|\alpha|<|\alpha+\beta|$
3. The relation between divided differences and forward differences is
   a) $\Delta^n f(x_0)=\dfrac{\Delta^n f(x_0)}{n!h^n}$ b) $\Delta^n f(x_0)=\dfrac{\Delta^{n-1} f(x_0)}{n!h^n}$
   c) $\Delta^{n+1} f(x_0)=\dfrac{\Delta^n f(x_0)}{n!h^n}$ d) $\Delta^n f(x_0)=\dfrac{\Delta^{n+1} f(x_0)}{(n+1)!\,h^{n+1}}$
4. The nth divided differences of a polynomial of nth degree are
   a) n b) n-1
   c) 0 d) Constant
5. If h = 1, $\Delta y_0=0.0244$, $\Delta^2 y_0=-0.0003$, $\Delta^3 y_0=0$ then the value of $\left(\dfrac{dy}{dx}\right)_{x=x_0}$ is
   a) 0.02455 b) 0.02275
   c) -0.0003 d) 0.0232
6. To evaluate $\int_4^{5.2}\log_e x\,dx$ using Trapezoidal, Simpson's 1/3 rd and Simpson's 3/8 th rule the proper choice of h will be
   a) 0.2 b) 0.3
   c) 0.4 d) 0.12
7. Given the equation $\dfrac{dy}{dx}=f(x,y),\ y(x_0)=y_0$, the value of $f(x_0,y_0)$ gives ___________
   a) the slope of the normal at (x_0,y_0) b) the slope of the tangent at (x_0,y_0)
   c) the value of y at (x_0,y_0). d) the slope of the tangent at (x_1,y_1)
8. To find y_{n+2} using Milne's method we need
   a) $y_{n-3}, y_{n-2}, y_{n-1}, y_n$ b) $y_{n-2}, y_{n-1}, y_n, y_{n+1}$
   c) $y_n, y_{n+1}, y_{n+2}, y_{n+3}$ d) $y_{n-4}, y_{n-3}, y_{n-2}, y_{n-1}$
9. To solve the equation $u_{xx}=au_t$, $\lambda=ka/h^2$, by Bender - Schmidt method, the formula is valid if
   a) $0\le\lambda\le 1$ b) $0\le\lambda\le 2$
   c) $0\le\lambda\le\frac{1}{2}$ d) $0\le\lambda\le\frac{3}{4}$
10. The equation $yu_{xx}+u_{yy}=0$ is hyperbolic in the region ___________
    a) y > 0 b) y < 0
    c) y = 0 d) $y^2 = 0$

PART B (10 x 2 = 20 Marks)

11. Is Gauss Seidel method applicable for solving non diagonally dominant system of equations? Justify.
12. If g(x) is continuous in [a, b] then under what condition the iterative method x = g(x) has a unique solution in [a, b] .
13. Find the quadratic polynomial that fits $y(x)=x^4$ at x = 0, 1, 2.
14. If Δ is the divided difference operator, then find $\Delta_{bc}^2\left(\dfrac{1}{a}\right)$.
15. By differentiating Newton's backward difference formula, find the first derivative of the function f(x).
16. Write any two practical situations where Simpsons rule is applicable.
17. Compare and contrast R-K method with Taylors series method.
18. Using Euler's method solve $y'=y+e^x$, $y(0)=0$, to find $y(0.2)$.

19. Write the implicit scheme for one dimensional heat equation $u_t = \alpha^2 u_{xx}$.

20. For what value of λ, is the explicit method of solving the hyperbolic equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \text{ stable, where } \lambda = c\frac{\nabla t}{\nabla x}?$$

PART C (5 x 14 = 70 Marks)

21. a) (i) Find a real root of cos x = 3x-1 by the method of false position, correct to three decimal places. (7)

(ii) Using Gauss Jordan method find the inverse of $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$ (7)

(OR)

b) (i) Find the positive root of the equation $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}e^{0.3x}$ using Newton-Raphson method. (7)

(ii) Solve by Gauss-Seidel method 4x + 2y +z =14; x + y + 8z = 20; x + 3y – z =10. (7)

22. a) The table gives the distance in nautical miles of the visible horizon for the given heights in feet above the earth's surface :

Height :	100	150	200	250	300	350	400
Distance :	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Find the distance when the height is 218 ft and 410 ft.

(OR)

b) (i) Determine f(x) as a polynomial in x for the following data. (7)

x :	- 4	-1	0	2	5
y :	1245	33	5	9	1335

(ii) The pressure p of wind corresponding to velocity v is given by the following data. Estimate p when v = 25. (7)

v :	10	20	30	40
p :	1.1	2	4.4	7.9

23. a) (i) Find $f'(x)$, $f''(x)$ at x =1.5 given (7)

x:	1.5	2.0	2.5	3.0	3.5	4.0
f(x):	3.375	7.0	13.625	24.0	38.875	59.0

(ii) Integrate numerically $\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta$ using Simpsons rule. (7)

(OR)

b) (i) Evaluate $\int_1^2 \int_1^2 \frac{dx\, dy}{x+y}$ by using Trapezoidal rule by taking 4 subintervals. (7)

(ii) Find $f'(10)$ from the following data. (7)

x:	3	5	11	27	34
f(x):	-13	23	899	17315	35606

24. a) (i) Using Runge – Kutta method of fourth order, find numerical solution at x = 0.8 for $\frac{dy}{dx} = \sqrt{x+y}$, $y(0.4) = 0.41$. Assume the step length as 0.2. (10)

(ii) If $y' = \frac{y-x}{y+x}; y(0) = 1$, find $y(0.02)$ by improved Euler's method. (4)

(OR)

b) (i) Given $y' = x^2 + y^2$, $y(0) = 1, y(-0.1) = 0.9088, y(0.1) = 1.1115$, $y(0.2) = 1.2530$ compute $y(0.3)$ by Milne's method. (7)

(ii) Find $y(0.1)$ and $y(0.2)$ using Taylor series method given $\frac{dy}{dx} = x^2 y - 1$, $y(0) = 1$. (7)

25. a) Solve the Laplace equation $\nabla^2 u = 0$ satisfying $u(0, y) = 0$, $u(3, y) = 3y + 9$, $u(x,0) = x^2$ and $u(x,3) = 2x^2$ by Leibmann's method, correct to two decimal places taking h=1.

(OR)

b) (i) Solve $y_{tt} = y_{xx}$ up to t = 0.5 with a spacing at 0.1 subject to $y(0,t) = 0,\ y(1,t) = 0,\ y_t(x,0) = 0$ *and* $y(x,0) = 10 + x(1-x)$. (7)

(ii) Solve $u_{xx} = u_t$ given $u(0,t) = 0,\ u(4,t) = 0, u(x,0) = x(4-x)$ assuming h = k = 1 up to t = 5. (7)

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