Register Number: ……………………..

B.E DEGREE EXAMINATIONS: APRIL/MAY 2014

(Regulation 2009)

Seventh Semester

COMPUTER SCIENCE ENGINEERING

MAT112: Partial Differential Equations and Their Solution Methodologies

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. Solution of the equation $\frac{\partial z}{\partial x} = 0$ is
   a) x = 0 b) $z = x f(y) + \emptyset(y)$
   c) z = f(x) d) $z = \emptyset(y)$

2. The particular integral of the equation $[(D-1)^2 - D'^2]z = e^{x-y}$ is
   a) e^{x-y} b) xe^{x-y}
   c) $-e^{x-y}$ d) $x^2 e^{x-y}$

3. The canonical form of the wave equation $u_{tt} = c^2 u_{xx}$ when $\xi = x - ct$, $\eta = x + ct$ is
   a) $u_{\eta\eta} = 0$ b) $u_{\xi\eta} = 0$
   c) $u_{\xi\xi} = 0$ d) $u_{\xi\eta} = c^2$

4. The type of partial differential equation $4\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} - 6\frac{\partial u}{\partial x} - 8\frac{\partial u}{\partial y} - 16u = 0.$ is
   a) Parabolic b) hyperbolic
   c) elliptic d) Both hyperbolic and elliptic

5. Choose a proper solution of steady state heat flow in two dimensions, if the non-zero boundary values is prescribed on y=a
   a) $u(x,y) = (Ae^{px} + Be^{-px})(C \cos py + D \sin py)$
   b) $u(x,y) = (Ae^{py} + Be^{-py})(C \cos px + D \sin px)$
   c) $u(x,y) = (Ax + B)(Cy + D)$
   d) $u(x,y) = (C \cos py + D \sin py)(C \cos px + D \sin px)$

6. The partial differential equation that represents steady state heat flow in polar coordinates is
   a) $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$
   b) $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$
   c) $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^3}\frac{\partial^2 u}{\partial \theta^2} = 0$
   d) $\frac{\partial^2 u}{\partial \theta^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial r^2} = 0$

7. Laplace transform of Heavyside's unit step function is
   a) e^{-as} b) $\frac{e^{-s}}{s}$
   c) $\frac{e^{-as}}{s}$ d) $\frac{1}{s}$

8. $L(\partial u/\partial t)$ is
   a) $s\bar{u} - u(o,t)$ b) $s\bar{u} - u(x,o)$
   c) $s\bar{u} + u(x,o)$ d) $s\bar{u} - u_t(x,o)$

9. $F\left[e^{-|x|}\right]$ is
   a) does not exist b) $\sqrt{\frac{2}{\pi}}\ \frac{1}{s^2+1}$
   c) $\sqrt{\frac{2}{\pi}}\frac{s}{s^2+1}$ d) $\sqrt{\frac{2}{\pi}}\ \frac{1}{s^2-1}$

10. The one dimensional diffusion equation is
    a) $\frac{\partial u}{\partial x} = K\frac{\partial^2 u}{\partial t^2}$ b) $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial t}$
    c) $k\frac{\partial u}{\partial x} = \frac{\partial u}{\partial t}$ d) $K\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$

PART B (10 x 2 = 20 Marks)

11. Form the PDE by eliminating the arbitrary constants a and b from $z = (x+a)^2 + (y+b)^2$.
12. Find the complete integral of $z = px + qy + \sqrt{pq}$ where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$.
13. Classify the equation $3\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 6\frac{\partial^2 u}{\partial y^2} - 2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} - u = 0.$
14. Why we need to transform PDE to canonical form?
15. Write any two solutions of the Laplace equation in Cartesian coordinates obtained by the method of separation of variables.
16. How will you choose the proper solution for the steady state heat flow in plate that does not contain the pole?
17. Find the Laplace transform of $\frac{\partial u}{\partial t}$ where u is a function of two variables x and t.
18. Find the inverse Laplace transform of $\frac{e^{-2s}}{s^2+4}$.
19. Find the Fourier Cosine transform of e^{-2x}.
20. Find the Fourier Sine transform of f(x) = 1 in $(0, l)$.

PART C (5 x 14 = 70 Marks)

21. a) (i) Form the PDE by eliminating the arbitrary functions for $z = x^2 f(y) + y^2 g(x)$. (7)

(ii) Solve: $z = px + qy + \sqrt{p^2 + q^2 + 16}$ (7)

(OR)

b) (i) Solve: $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$ (6)

(ii) Solve: $(D^3 + D^2D' - DD'^2 - D'^3)z = e^{2x+y} + \cos(x+y)$ (8)

22. a) Reduce the relation $3u_{xx} + 10u_{xy} + 3u_{yy} = 0$ to a canonical form and solve it.

(OR)

b) Reduce the equation $u_{xx} - 2\sin x\, u_{xy} - cos^2 x u_{yy} - \cos x\, u_y = 0$ to a canonical form and solve it.

23. a) A square plate is bounded by the lines x = 0, y = 0, x = 20 and y = 20. Its faces are insulated. The temperature along the upper horizontal edge is given by

u(x, 20) = x (20 - x) when 0 < x < 20

While the other three edges are kept at $0^0 c$. Find the steady state temperature of the plate.

(OR)

b) Find the steady-state temperature function $u(r, \theta)$ in a semi-circular plate of radius a, insulated on both the faces with its curved boundary kept at a constant temperature U_0 and its bounding diameter kept at zero temperature.

24. a) Using the Laplace transform method, find the solution $\theta(x, t)$ of one dimensional diffusion equation $\frac{\partial^2\theta}{\partial x^2} = \frac{1}{k}\frac{\partial\theta}{\partial t}$, $0 \le x \le \pi, t > 0$ satisfying the boundary conditions $\theta(0, t) = 1 - e^{-t}, t > 0$

$$\theta(\pi, t) = 0,\ t > 0$$

$$\theta(x, 0) = 0, t = 0, 0 \le x \le \pi$$

(OR)

b) Using the Laplace transform method, solve the IBVP described by

PDE : $u_{xx} = u_{tt}$; $0 < x < 1, t > 0$

BCs : $u(0, t) = u(1, t) = 0,\ t > 0$

ICs : $u(x, o) = \sin \pi x$

$u_t(x, 0) = -\sin \pi x,\ 0 < x < 1.$

25. a) Solve the heat conduction problem described by

$$k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t},\ 0 < x < \infty, \quad t > 0$$

$$u(0, t) = u_o, t \ge 0$$

$$u(x, 0) = 0, \quad 0 < x < \infty,$$

u and $\frac{\partial u}{\partial x}$ $both\ tend\ to\ zero\ as\ x \to \infty.$

(OR)

b) Using Transform method, find the steady-state temperature distribution $u(x, y)$ in a long square bar of side π with one face maintained at constant temperature u_0 and the other faces at zero temperature.

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