Register Number: ……………………..

MCA DEGREE EXAMINATIONS: APRIL/MAY 2014

(Regulations 2009)

Second Semester

MASTER OF COMPUTER APPLICATIONS

MAT509: Mathematical Foundations of Computer Science

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART B (10 x 2 = 20 Marks)

1. State Cayley-Hamilton Theorem.
2. Find the eigen values of A^{-1} , if A = $\begin{pmatrix} 2 & 0 & 0 \\ 3 & 6 & 0 \\ 5 & 1 & 3 \end{pmatrix}$.
3. Define anti symmetric.
4. Let A =$\{a, b, c\}$. Find all the partitions of A.
5. Write the symbolic form of " Sam is poor and Ram is intelligent " using predicate.
6. Construct the truth table for $P \wedge (P \vee \neg Q)$.
7. Define ambiguous grammar.
8. Define phrase structure grammar.
9. Define NDFA.
10. Construct the state diagram for the automatan given by M= $(\{q_0, q_1, q_2\}, \{a, b\}, \delta, q_0, \{q_2\})$ when δ is $\delta(q_0, a) = q_1$, $\delta(q_1, a) = q_1$, $\delta(q_2, a) = q_1$, $\delta(q_0, b) = q_2$, $\delta(q_1, b) = q_2$, $\delta(q_2, b) = q_1$.

PART C (5 x 16 = 80 Marks)

11. a) (i) Test for the consistency of the following system of equations and solve them, if consistent. 3x + y+ z = 8 ; -x + y-2z = -5 ; x + y + z =6; -2x +2y -3z = -7. (8)

(ii) Find the eigen values and eigen vectors of (adj A), given that the matrix (8)

A = $\begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{pmatrix}$.

(OR)

b) (i) Verify Cayley – Hamilton theorem for the matrix A=$\begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$ and also use it to find A^{-1}. (8)

(ii) Find the condition satisfied by a, b, c so that the following system of equations may have a solution x + 2y -3z = a ; 3x – y + 2z = b ; x – 5y + 8z = c. (8)

12. a) (i) How many integers are between 1 and 250 that are divisible by any of the integers 2,3,5 and 7. (8)

(ii) Let A,B,C be sets such that A∪B = A∪C and A∩B = A∩C, show that B = C. (8)

(OR)

b) (i) Show that f : R - {3} → $R - \{1\}$ given by $f(x) = \frac{x-2}{x-3}$ is a bijection. (8)

(ii) Let X = {1,2,3 ... 7 } and R = $\{(x, y)/x - y \text{ is divisible by } 3\}$. Show that R is an reflexive, symmetric and transitive. (8)

13. a) (i) Show that $((p \wedge q) \wedge (p \to r) \wedge (q \to r)) \to r$ is a tautology (8)

(ii) Obtain the principal disjunctive normal form of $(P \wedge Q) \vee (\neg P \wedge R)$. (8)

(OR)

b) (i) Prove that $P \to Q, Q \to \neg R, R, P \vee (J \wedge S) \Longrightarrow J \wedge S$. (8)

(ii) Determine the validity of the following argument: (8)

If 7 is a prime number, then 7 does not divide 35. 7 divides 35. Therefore 7 is not a prime number.

14. a) (i) Construct a grammar G for the language L(G) =$\{a^n ba^m ; n, m \geq 1\}$ (8)

(ii) Define the types of phrase structure grammars (8)

(OR)

b) (i) State and prove pumping lemma (8)

(ii) Find the language generated by the grammar G=$\{(S,A),(a,b),S,P\}$ where P consists of the production $\{S \longrightarrow aA, S \rightarrow bS, S \rightarrow a, S \rightarrow b, A \rightarrow bA, A \rightarrow bS, A \rightarrow b\}$ (8)

15. a) (i) Design a finite state automata which accept the string 1101. (8)

(ii) Construct a deterministic finite automation equivalent to (8)

M =$(\{q_{0,} q_{0,} q_{2}, q_3\},\{0,1\},\delta,q_{0,}\{q_3\})$. δ is given in following table

δ^1	a	b
q_0	$q_{0,}q_1$	q_0
q_1	q_2	q_1
q_2	q_3	q_3
q_3	-	q_2

(OR)

b) Let M = $(\{q_{0,} q_{1,} q_{2}, q_3\},\{a,b\},\delta,q_{0,}\{q_1\})$ where δ *is* given by $\delta(q_{0,}a) = q_1$, $\delta(q_{0,}b) = q_2$, $\delta(q_{1,}a) = q_3$, $\delta(q_{1,}b) = q_0$, $\delta(q_{2,}a) = q_2$, $\delta(q_{2,}b) = q_2$, $\delta(q_{3,}a) = q_2$, $\delta(q_{3,}b) = q_2$.

a). Represent M by its state table

b). Represent M by its state diagram.

c). Which of the following string are accepted by M?

ababab, aabbaa, abba

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