Register Number: ……………………..

B.E DEGREE EXAMINATIONS: MAY/JUNE 2014

(Regulation 2007)

First Semester

MECHANICAL ENGINEERING

U07MA101**:** Mathematics – I

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. The eigen values of the matrix $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 3 & 0 \\ 1 & 1 & 2 \end{pmatrix}$ are

    a) 1, -2, 7 b) -1, -3, -2
    c) 1, 2, 3 d) -1, -3, 10

2. If A is an orthogonal matrices, then

    a) $A^{-1} = A^T$ b) $|A| = \pm 1$
    c) $AA^T = A^T A = I$ d) All the above

3. The direction ratios of the line joining the points (2, 3, 5) and (1, 4, -7) is

    a) (-1, 1, -12) b) (3, 7, -2)
    c) (-1,-1,12) d) (1, 7, -2)

4. The equation of the plane through the point (1, 2, 3) and parallel to the plane $3x+4y+5z=0$ is

    a) $3x+4y+5z+26=0$ b) $3x+4y+5z-26=0$
    c) $3x+4y+5z-6=0$ d) $x+2y+3z=0$

5. The reciprocal of the radius of curvature is called

    a) curvature b) evolute
    c) envelope d) involute

6. The circle of curvature of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at $\left(\frac{a}{4}, \frac{a}{4}\right)$ whose centre and radius of curvature are $\left(\frac{3a}{4}, \frac{3a}{4}\right)$ and $\frac{a}{\sqrt{2}}$ respectively is

    a) $\left(x-\frac{3a}{4}\right)^2+\left(y-\frac{3a}{4}\right)^2=\frac{a}{\sqrt{2}}$ b) $\left(x-\frac{3a}{4}\right)^2-\left(y-\frac{3a}{4}\right)^2=\frac{a^2}{2}$
    c) $\left(x-\frac{3a}{4}\right)^2+\left(y-\frac{3a}{4}\right)^2=\frac{a^2}{2}$ d) $\left(x-\frac{a}{4}\right)^2+\left(y-\frac{a}{4}\right)^2=\frac{a^2}{2}$

7. If $\Delta > 0$ and $A > 0$, then the function f (x, y) attains at (a, b) is

    a) maximum b) minimum
    c) neither maximum nor minimum d) saddle point

8. If $u = xe^y \sin z,\ v = xe^y \cos z,\ w = x^2 e^{2y}$ are functionally dependent, then the relationship between them is

    a) $u^2 = v^2 + w$ b) $u^2 - v^2 = w$
    c) $u^2 + v^2 = w^2$ d) $u^2 + v^2 = w$

9. The complementary function of $(D+3)^2 y = 3e^{-3x}$ is

    a) $Ae^{-3x} + Be^{3x}$ b) $(Ax+B)e^{-3x}$
    c) $A\cos 3x + B\sin 3x$ d) $Ae^{-3x} + Be^{-3x}$

10. If $f(x) = e^x$ *and* $g(x) = e^{-x}$, then the value of *Wronskian* is

    a) 1 b) -1
    c) -2 d) 2

PART B (10 x 2 = 20 Marks)

11. Define eigen values and eigen vectors.
12. Show that matrix is singular if and only if one of its eigen values is zero.
13. Find the angle between the lines $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-1}{-1}$ and $\frac{x}{3}=\frac{y+1}{-1}=\frac{z+2}{1}$.
14. Define Sphere.
15. Define evolute.
16. Find the envelope of the family of straight lines $y = mx + am^2$ where m is the parameter.
17. Find the maximum or minimum of the function $y^2 + 4xy + 3x^2 + x^3$.
18. If $u = x^2 y z^3$ where $x = t,\ y = t^3,\ z = e^{2t}$, find $\frac{du}{dt}$.
19. Convert the variable coefficient of the differential equation $x^2 \frac{d^2y}{dx^2} + 8x\frac{dy}{dx} + 13y = \log x$ into constant coefficient of the differential equation.
20. Find the P. I of $(D^2+1)y = \sin x$.

PART C (5 x 14 = 70 Marks)

21. a) (i) Show that (1, 3, 2) (5, -2, 1) and (-7, 13, 4) are linearly dependent. (4)

(ii) Using Cayley – Hamilton theorem, find the inverse of the matrix $\begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{pmatrix}$. (10)

(OR)

b) Reduce the quadratic form $2x^2 + 5y^2 + 3z^2 + 4xy$ to canonical form by orthogonal reduction and find the rank, index, signature and nature of the quadratic form.

22. a) (i) Find the shortest distance between the lines $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$. (7)

(ii) Show that the spheres $x^2 + y^2 + z^2 + 6y + 2z + 8 = 0$ and $x^2 + y^2 + z^2 + 6x + 8y + 4z + 20 = 0$ cut each other orthogonally. Find their plane of intersection. (7)

(OR)

b) (i) Find the equation to the straight line which passes through the point (1, 3, 5) and parallel to the plane $x + y + z + 2 = 0$ and perpendicular to the line $2x + y = 0, x - z + 5 = 0$. (7)

(ii) Find the equation of the sphere having the circle $x^2 + y^2 + z^2 - 10y + 4z - 8 = 0, x + y + z = 0$ as a great circle. (7)

23. a) (i) Show that the radius of curvature of $xy^2 = a^2(a-x)$ at (a, 0) is $\frac{a}{2}$. (7)

(ii) Find the envelope of $\frac{x}{a} + \frac{y}{b} = 1$ where $a^n + b^n = c^n$. (7)

(OR)

b) (i) Find the centre of curvature at θ on $x = a(\theta + \sin\theta), \quad y = a(1 - \cos\theta)$. (7)

(ii) Considering the evolute as the envelope of normals, find the evolute of the parabola $x^2 = 4ay$. (7)

24. a) (i) Expand $x^2 y + 3y - 2$ in powers of (x – 1) and (y + 2) up to 3rd degree terms. (7)

(ii) If $x = r\sin\theta\cos\phi, \quad y = r\sin\theta\sin\phi, \quad z = r\cos\phi$ find $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$. (7)

(OR)

b) (i) If $z = f(x, y)$ where $x = u^2 - v^2, \ y = 2uv$, prove $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{1}{4(u^2+v^2)}\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right)$. (7)

(ii) Show that of all rectangular parallelopipeds of given volume, the cube has the least surface area. (7)

25. a) (i) Solve $y'' + 7y' - 8y = e^{2x}$ using the method of variation of parameters. (7)

(ii) Solve $(D^2 - 7D + 10)y = e^{2x} + 10x^2$. (7)

(OR)

b) Solve $\frac{dx}{dt} - y = t, \quad \frac{dy}{dt} + x = 1$.

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