

B.E DEGREE EXAMINATIONS: MAY/JUNE 2014

(Regulation 2013)

Second Semester

U13MAT201: ENGINEERING MATHEMATICS -II

(Common to AE/ECE)

Time: Three Hours

Maximum Marks: 100

**Answer all the Questions:-
PART A (10 x 1 = 10 Marks)**

- The value of $\int_0^1 \int_0^2 \int_0^3 xyz \, dx \, dy \, dz$ is
 - $\frac{5}{2}$
 - $\frac{9}{4}$
 - $\frac{2}{5}$
 - $\frac{9}{2}$
- The volume of the sphere $x^2 + y^2 + z^2 = a^2$ is
- If $\mathbf{A} = \text{curl } \mathbf{F}$, then $\iint_S \mathbf{A} \cdot d\mathbf{S} = \dots\dots\dots$ where S is any closed surface
- If \mathbf{r} is the position vector of the point (x,y,z) with respect to origin then $\text{div } \mathbf{r}$ is
 - 2
 - 1
 - 3
 - 0
- Cauchy – Riemann equations for the function $f(z) = u(x, y) + i v(x, y)$ is
 - $u_x = v_y ; u_y = -v_x$
 - $u_x = u_y , v_x = -v_y$
 - $u_y = v_x , u_x = -v_y$
 - $u_x = v_y , u_y = v_x$
- The function $f(z) = \frac{1}{z^2 + 1}$ is analytic everywhere except at
- $f(z) = e^{1/z}$, $z = 0$ is singularity
- C is a simple closed curve and a is a point lying outside C, then $\int_C \frac{dz}{z-a}$ is
 - $2\pi i$
 - $-2\pi i$
 - 0
 - 2a
- $L[t^3 e^{-t}]$ is

- $\frac{6}{s}$
- $\frac{3}{s}$
- $\frac{3}{(s+1)}$
- $\frac{6}{(s+1)^4}$

10. If $L[f(t)] = F(s)$, then $L[f(at)]$ is

PART B (10 x 2 = 20 Marks)

- Evaluate $\int_1^2 \int_1^3 xy^2 \, dx \, dy$.
- Find the area of the circle of radius 'a', using polar co ordinates.
- In what direction from (3,1,-2) is the directional derivative of $\phi = x^2 y^2 z^4$ maximum?
- State Green's theorem on xy plane.
- If $f(z)$ is analytic, show that $f(z)$ is constant if $f'(z) = 0$ everywhere.
- Define conformal mapping.
- State Cauchy's integral theorem.
- Find the residue of $f(z)$ at the simple pole where $f(z) = \frac{z^2}{(z-1)^2(z+2)}$.
- Write the existence conditions of Laplace transform.
- Find the inverse Laplace transform of $\log \frac{s+1}{s-1}$.

PART C (5 x 14 = 70 Marks)

Q. No. 21 is compulsory

- Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ (7)
 - Change the order of integration and evaluate $\int_0^{1-2x} \int_{x^2}^x xy \, dy \, dx$ (7)
- Prove $\vec{F} = (x+2y+4z)\vec{i} + (2x-3y-z)\vec{j} + (4x-y+2z)\vec{k}$ is irrotational. Hence find ϕ such that $\vec{F} = \nabla \phi$. (7)
 - Prove that area bounded by a simple closed curve C is given by $\frac{1}{2} \int_C (x \, dy - y \, dx)$. (7)

(OR)

b) (i) Verify Gauss divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ taken over the rectangular parallelepiped $0 \leq x \leq a$ $0 \leq y \leq b$ $0 \leq z \leq c$.

b) (i) Using convolution theorem find $L^{-1}\left[\frac{s}{(s^2 + a^2)^2}\right]$ (7)

23. a) (i) Find the analytic function whose real part is $u = \frac{\sin 2x}{\cosh 2y - \cos 2x}$ (7)

(ii) Using Laplace transform, solve (7)

$$\frac{d^2 y}{dt^2} + 4 \frac{dy}{dt} + 3y = e^t \quad \text{given that } y(0) = 0 \text{ and } y'(0) = 0$$

(ii) If $f(z) = u + iv$ is an analytic function then prove that $u(x,y) = C_1$ and $v(x,y) = C_2$ cut each other orthogonally where C_1 and C_2 are constants. (7)

(OR)

b) (i) If $f(z)$ is a regular function of z , prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$ (7)

(ii) Find the bilinear transformation that maps $1, i, -1$ of Z -plane onto $0, 1, \infty$ of the w -plane. (7)

24. a) (i) Evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$ where C is the circle $|z|=2$ (7)

(ii) Find the Laurent's series expansion of (7)

$$f(z) = \frac{7z-2}{z(z+1)(z-2)} \quad \text{for } 1 < |z+1| < 3$$

(OR)

b) Using contour integration evaluate $\int_0^{2\pi} \frac{d\theta}{13 + 5 \sin \theta}$

25. a) (i) Find the Laplace transform of (i) $\frac{1-e^t}{t}$ (ii) $t \sin 3t$ (7)

(ii) Find the Laplace transform of the function $f(t)$ with the period $2\pi/\omega$, where (7)

$$f(t) = \begin{cases} \sin \omega t & \text{for } 0 < t < \frac{\pi}{\omega} \\ 0 & \text{for } \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$$

(OR)