Register Number: ……………………..

B.E/B.TECH DEGREE EXAMINATIONS: MAY/JUNE 2014

(Regulation 2013)

Second Semester

U13MAT201: ENGINEERING MATHEMATICS -II

(Common to MCE/Mech/IT)

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. The value of $\int_0^1\int_0^2\int_0^3 xyz\,dx\,dy\,dz$ is

   a) $\frac{9}{4}$ b) $\frac{5}{2}$

   c) $\frac{2}{5}$ d) $\frac{9}{2}$

2. The volume of the sphere $x^2 + y^2 + z^2 = a^2$ is ………………

3. If **A**=curl **F**, then $\iint_S \vec{A}.d\vec{s} =$ ……………….. where S is any closed surface

4. If **r** is the position vector of the point (x,y,z) with respect to origin then div **r** is

   a) 2 b) 1

   c) 3 d) 0

5. Cauchy – Riemann equations for the function f(z) = u (x, y) + i v(x, y) is

   a) $u_x = v_y$; $u_y = -v_x$ b) $u_y = v_x$, $u_x = -v_y$

   c) $u_x = u_y$, $v_x = -v_y$ d) $u_x = v_y$, $u_y = v_x$

6. The function $f(z) = \frac{1}{z^2+1}$ is analytic everywhere except at …………….

7. $f(z) = e^{1/z}$, z = 0 is ………………….. singularity

8. C is a simple closed curve and a is a point lying outside C ,then $\int_C \frac{dz}{z-a}$ is

   a) 2πi b) -2πi

   c) 0 d) 2a

9. $L\left[t^3 e^{-t}\right]$ is

   a) $\frac{3}{(s+1)}$ b) $\frac{3}{s}$

   c) $\frac{6}{s}$ d) $\frac{6}{(s+1)^4}$

10. If $L\left[f(t)\right]$ = F(s), then $L\left[f(at)\right]$ is …………………..

PART B (10 x 2 = 20 Marks)

11. Evaluate $\int_1^2\int_1^3 x\,y^2\,dx\,dy$.

12. Find the area of the circle of radius ‘a’, using polar co ordinates.

13. In what direction from (3,1,-2) is the directional derivative of $\phi = x^2y^2z^4$ maximum?

14. State Green’s theorem on xy plane.

15. If f(z) is analytic , show that f(z) is constant if $f'(z) = 0$ everywhere.

16. Define conformal mapping.

17. State Cauchy’s integral theorem.

18. Find the residue of f(z) at the simple pole where f(z) = $\frac{z^2}{(z-1)^2(z+2)}$.

19. Write the existence conditions of Laplace transform.

20. Find the inverse Laplace transform of $\log\frac{s+1}{s-1}$.

PART C (5 x 14 = 70 Marks)

Q. No. 21 is compulsory

21. (i) Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ (7)

    (ii) Change the order of integration and evaluate $\int_0^1\int_{x^2}^{2-x} xy\,dy\,dx$ (7)

22. a) (i) Prove $\vec{F} = (x+2y+4z)\vec{i} + (2x-3y-z)\vec{j} + (4x-y+2z)\vec{k}$ is irrotational. Hence find ϕ *such that* $\vec{F} = \nabla\phi$. (7)

    (ii) Prove that area bounded by a simple closed curve C is given by $\frac{1}{2}\int_C (xdy - ydx)$. (7)

(OR)

b) (i) Verify Gauss divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ taken over the rectangular parallelepiped $0 \le x \le a \quad 0 \le y \le b \quad 0 \le z \le c$.

23. a) (i) Find the analytic function whose real part is $u = \dfrac{\sin 2x}{\cosh 2y - \cos 2x}$ (7)

(ii) If f(z) =u+iv is an analytic function then prove that u(x,y) = C_1 and v(x,y) = C_2 cut each other orthogonally where C_1 and C_2 are constants. (7)

(OR)

b) (i) If f(z) is a regular function of z, prove that $\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f^1(z)|^2$ (7)

(ii) Find the bilinear transformation that maps 1,i,-1 of Z-plane onto 0,1,∞ of the w-plane. (7)

24. a) (i) Evaluate $\int_C \dfrac{e^{2z}}{(z+1)^4} dz$ where C is the circle $|z| = 2$ (7)

(ii) Find the Laurent's series expansion of (7)

$$f(z) = \frac{7z-2}{z(z+1)(z-2)} \quad for\ 1 < |z+1| < 3$$

(OR)

b) Using contour integration evaluate $\displaystyle\int_0^{2\pi} \frac{d\theta}{13 + 5\sin\theta}$

25. a) (i) Find the Laplace transform of $(i)\ \dfrac{1-e^t}{t} \quad (ii)\ t\sin 3t$ (7)

(ii) Find the Laplace transform of the function f(t) with the period 2π/ω, where (7)

$$f(t) = \begin{cases} \sin\omega t & for\ 0 < t < \dfrac{\pi}{\omega} \\ 0 & for\ \dfrac{\pi}{\varpi} < t < \dfrac{2\pi}{\varpi} \end{cases}$$

(OR)

b) (i) Using convolution theorem find $L^{-1}\left[\dfrac{s}{(s^2 + a^2)^2}\right]$ (7)

(ii) Using Laplace transform , solve (7)

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = e^t \quad given\ that\ y(0) = 0\ and\ y^1(0) = 0$$

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