KUMARAGURU
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Register Number: ……………………..

B.E/ B.TECH DEGREE EXAMINATIONS: DEC 2014

(Regulation 2009)

First Semester

MAT101: ENGINEERING MATHEMATICS- I

(Common to all Branches)

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. If 1 and 5 are the eigen values of $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$, then the eigen values of $2A^2$ are

   a) 1 , 25   b) 2 , 10

   c) 2 , 50   d) 1 , 50

2. If A is a singular matrix, then the corresponding quadratic form is known as

   a) Singular   b) No- singular

   c) Positive definite   d) Negative definite

3. Two straight lines which do not lie in the same plane is called as

   a) Parallel lines   b) Skew lines

   c) Plane section   d) Coplanar lines

4. The centre of the sphere $2(x^2 + y^2 + z^2) + 6x - 6y + 8z + 9 = 0$ *is*

   a) (3 , - 3, 4)   b) (-3, 3, -4)

   c) $\left(-\frac{3}{2}, \frac{3}{2}, -2\right)$ d) $\left(\frac{3}{2}, \frac{3}{2}, 2\right)$

5. The radius of curvature of $x^2 + y^2 - 2x - 4y - 2 = 0$ is

   a) $\sqrt{7}$ b) $1/\sqrt{7}$

   c) 7   d) 3

6. The locus of intersection of consecutive members of a family of curves is called as

a) Evolute b) Envelope

c) Involute d) Curvature

7. A function f (x , y) is said to have a relative maximum at (a , b) only when

a) $f\ (a,b) > f\ (a+h, b+k)$ b) $f\ (a,b) < f\ (a+h, b+k)$

c) $f\ (a,b) = f\ (a+h, b+k)$ d) $f\ (a,b) = f\ (a^2, b^2)$

8. If u and v are functionally dependent with respect to x and y, then

a) $\frac{\partial(u,v)}{\partial(x,y)} = 0$ b) $\frac{\partial(u,v)}{\partial(x,y)} = 1$

c) $\frac{\partial(x,y)}{\partial(u,v)} = 0$ d) $\frac{\partial(x,y)}{\partial(u,v)} = 1$

9. The complementary function of $(D^2 + 6D + 9)y = 0$

a) $(Ax + B)\, e^{-3x}$ b) $(A + B)\, e^{-3x}$

c) $Ae^{3x} + B\, e^{-3x}$ d) $(Ax + B)\, e^{3x}$

10. The differential equation of constant coefficient for the equation $x^2 D^2 + x D + 1 = 0$ *is*

a) $\theta^2 + \theta + 1 = 0$ b) $\theta^2 + 1 = 0$

c) $\theta^2 = 1$ d) $\theta^2 + \theta = 0$

PART B (10 x 2 = 20 Marks)

11. Two eigen values of the matrix $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ are equal to 1 each. Find the third eigen value.

12. Calculate the index and signature of the quadratic form $x^2 + 2y^2 - 3z^2$.

13. Establish the distance between the parallel planes $2x - 3y + 6z + 12 = 0$ *and* $6x - 9y + 18z - 6 = 0$.

14. Identify the angle between the lines $\frac{x}{2} = \frac{y}{-2} = \frac{z}{1}$ *and* $\frac{x-4}{2} = \frac{y-5}{1} = \frac{z+6}{2}$.

15. Determine the curvature of $y = e^x \; at \; x = 0$.

16. Construct the envelope of the family of curves $y = mx + \frac{a}{m}$, *m being the parameter*.

17. Show du int*erms of* $dx \; and \; dy \; from \; u = x\,y \log(x\,y)$.

18. Analyze the nature of the stationary point (1 , 1) of the function f (x, y) if

$f_{xx} = 6xy^3;\; f_{xy} = 9x^2y^2 \; and \; f_{yy} = 6x^3y.$

19. Form an equation of constant coefficient for

$(2x+3)^2 D^2 y - 2(2x+3)Dy - 12y = 6x.$

20. Solve $(D^2+1)^2 y = 0.$

PART C (5 x 14 = 70 Marks)

21. a) (i) Verify Cayley – Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$ *and hence find* A^{-1}. (7)

(ii) Examine the eigen values and vectors of the matrix $A = \begin{pmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$. (7)

(OR)

b) Reduce the quadratic form $x^2 + 2y^2 + z^2 - 2xy + 2yz$ to the canonical form through an orthogonal transformation and hence find its nature.

22. a) (i) Construct the equation of the plane which passes through the line of intersection of the planes $x + 5y - 2z = 6 \; and \; 5x - 4y + 5z = 2$ and is parallel to the line joining the points (0, 3, 4) and (1, 2, 3). (7)

(ii) Build the equation of the sphere which passes through the circle $x^2 + y^2 + z^2 = 4 \; and \; x = 0$ and which is cut by the planes $x + 2y + 2z = 0$ in a circle of radius 4. (7)

(OR)

b) Decide the length and equation of the shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ *and* $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$

23. a) (i) Estimate the radius of curvature of at the point $\left(\frac{3a}{2},\frac{3a}{2}\right)$ *on the curve* $x^3+y^3=3\,a\,x\,y$. (7)

(ii) Formulate the envelope of the straight line $\frac{x}{a}+\frac{y}{b}=1$ where a and b are the parameters that are connected by the relation a + b = c. (7)

(OR)

b) Show that the evolute of the cycloid $x=a\,(\theta-\sin\theta)$ and $y=a\,(1-\cos\theta)$ is another cycloid.

24. a) (i) Compose the Taylor's expansion of $e^x \sin y$ near the point $(-1\ ,\ \frac{\pi}{4})$ up to the second degree terms. (7)

(ii) Determine the Jacobian of y_1, y_2, y_3 *with respect to* x_1, x_2, x_3 *if* $y_1=\frac{x_2\,x_3}{x_1}\ ;y_2=\frac{x_3\ x_1}{x_2}\ ;\ y_3=\frac{x_1\,x_2}{x_3}$. (7)

(OR)

b) Estimate the shortest and the longest distances from the point (1 , 2 , -1) to the sphere $x^2+y^2+z^2=24$.

25. a) (i) Solve $\left(D^2+5\,D+4\right)y=e^{-x}\sin 2\,x$ (7)

(ii) Solve $\left(x^2\ D^2-2\,x\,D-4\right)y=32\,(\log x)^2$ (7)

(OR)

b) Apply the Method of variation of parameters to solve $\left(D^2+a^2\right)y=\tan 2x$.

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