KUMARAGURU
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Register Number: ……………………..

**B.E DEGREE EXAMINATIONS: NOV/DEC 2014**

(Regulation 2009)

Second Semester

**MAT102: ENGINEERING MATHEMATICS II**

(Common to AERO/AUTO/CE/ECE/EEE/EIE/ME/MCT)

**Time: Three Hours** **Maximum Marks: 100**

**Answer all the Questions:-**

**PART A (10 x 1 = 10 Marks)**

1. The integral $\int_0^{\pi}\int_0^{sin\theta} drd\theta$ is equal to

a) 0 b) 1

c) 2 d) 3

2. Polar form of the integral $\int_0^a\int_y^a \frac{x}{x^2+y^2}dxdy$ is equal to

a) $\int_0^{\frac{\pi}{4}}\int_0^{asec\theta} cos\theta drd\theta$ b) $\int_0^{\frac{\pi}{4}}\int_0^{asec\theta} \frac{rcos\theta}{1+r^2}drd\theta$

c) $\int_0^{\frac{\pi}{2}}\int_0^{asec\theta} \frac{cos\theta}{r^2}drd\theta$ d) $\int_0^{\frac{\pi}{2}}\int_0^{asec\theta} \frac{rcos\theta}{1+r^2}drd\theta$

3. The maximum directional derivative of $\phi = 4xz^2$ at $(1,-2,-1)$ is

a) $8\vec{i} - 4\vec{k}$ b) $4\vec{i} - 8\vec{k}$

c) $8\vec{i} + 4\vec{k}$ d) $4\vec{i} + 8\vec{k}$

4. The line integral $\int_C \vec{r}.d\vec{r}$ where C is the line $y = x$ in the $xy$-plane from $(0,0)$ to $(2,2)$ is equal to

a) 0 b) 1

c) 2 d) 3

5. Analytic function with constant modulus is

a) not harmonic b) complex constant

c) infinite at the origin d) not conjugate harmonic

6. The critical point of the transformation $w = \frac{1}{z}$ is

a) $z = \frac{\pi}{2}$ b) $z = \pm 1$

c) $z = 0$ d) $z = \infty$

7. If $f(a) = \int_{|z|=2} \frac{5z^2+7z+2}{Z-a} dz$, then $f(3)$ is equal to

a) 0 b) $-2\pi i$

c) $\frac{i\pi}{2}$ d) $4\pi i$

8. The residue of $f(z) = \frac{z^2}{(z-1)(z+2)^2}$ at its isolated singularity $z = 1$ is equal to

a) $-\frac{2}{9}$ b) $\frac{2}{9}$

c) $-\frac{1}{9}$ d) $\frac{1}{9}$

9. If $L\{f(t)\} = \frac{1}{s(s^2+4)}$ , then $f(\infty)$ is equal to

a) 0 b) $\frac{1}{4}$

c) $-\frac{1}{4}$ d) $\infty$

10. $L\left\{\int_0^t e^{-u} \sin(t-u)\, du\right\}$ is equal to

a) $\frac{s}{(s-1)(s^2+1)}$ b) $\frac{s}{(s+1)(s^2+1)}$

c) $\frac{1}{(s+1)(s^2+1)}$ d) $\frac{1}{(s-1)(s^2+1)}$

**PART B (10 x 2 = 20 Marks)**

11. Evaluate $\int_0^1 \int_0^2 \int_0^3 xyzdzdydx$.

12. Evaluate $\iiint_V \, dV$ where $V$ is enclosing a volume of the sphere given by $x^2 + y^2 + z^2 = a^2$.

13. If $\vec{F} = (2x - 5y)\vec{i} + (x + \lambda y)\vec{j} + (3x - z)\vec{k}$ is solenoidal, find $\lambda$ .

14. State Stoke's theorem in a plane.

15. Verify $e^x cosy$ is harmonic or not.

16. Find the invariant points of the bilinear transformation $w = \frac{2z+6}{z+7}$.

17. Expand $\frac{e^z}{(z-1)^2}$ in the region $0 < |z-1| < 1$.

18. Evaluate $\int_{|z|=1} \frac{e^{-z}}{z^2} dz$.

19. Find $L\left\{\cos\frac{t}{a}\right\}$.

20. Find $L^{-1}\{tan^{-1}(s)\}$.

**PART C (5 x 14 = 70 Marks)**

21. a) (i) Evaluate $\iint_R xydxdy$ where $R$ is the region bounded by the line $x+2y=2$ lying in the first quadrant. (7)

(ii) Evaluate $\int_0^{log2}\int_0^{x}\int_0^{x+y} e^{x+y+z}\,dxdydz$ (7)

**(OR)**

b) (i) Change the order of integration in $\int_0^a\int_{a-y}^{\sqrt{a^2-y^2}} ydxdy$ and evaluate it. (7)

(ii) Evaluate $\iiint_V \frac{dxdydz}{x^2+y^2+z^2}$ taken throughout the volume of the sphere $x^2+y^2+z^2=a^2$. (7)

22. a) (i) Find the values of the constants $a,b,c$ so that $\vec{F}=(axy-bz^3)\vec{\imath}+(3x^2-cz)\vec{\jmath}+(3xz^2-y)\vec{k}$ may be irrotational and hence find its scalar potential. (7)

(ii) Verify Stoke's theorem for $\vec{F}=-y\vec{\imath}+2yz\vec{\jmath}+y^2\vec{k}$ where $S$ is the upperhalf of the sphere $x^2+y^2+z^2=a^2$ and C is the circular boundary on the $xy$-plane. (7)

**(OR)**

b) (i) Find the angle between the normals to the surface $xy=z^2$ at the points $(-2,-2,2)$ and $(1,9,-3)$. (7)

(ii) Verify Divergence theorem for $\vec{F}(2x-z)\vec{\imath}+x^2y\vec{\jmath}-\boldsymbol{x}z^2\vec{k}$ where $S$ is the closed surface of the cube formed by $x=0,x=1,y=0,y=1,z=0$ and $z=1$. (7)

23. a) (i) Find the analytic function $f(z)=u+iv$ whose real part is given by $u=e^x(xsiny+ycosy)$. (7)

(ii) Find the bilinear map which maps the points $z=0,1,\infty$ into the points $w=i,1,-i$ respectively. (7)

**(OR)**

b) (i) If $f(z)=u+iv$ is an analytic function of $z=x+iy$, prove that $\nabla^2\{|u|^p\}=p(p-1)|u|^{p-2}|f'(z)|^2$. (7)

(ii) Find the image of the circle $|z-3| = 5$ under the transformation $w = \frac{1}{z}$. (7)

24. a) (i) Using Cauchy's integral formula, evaluate $\int_C \frac{z+1}{z^3-2z^2} dz$ where $C$ is the circle $|z-2-i| = 2$. (7)

(ii) Evaluate $\int_0^{2\pi} \frac{cos2\theta}{5+4cos\theta} d\theta$ using contour integration. (7)

**(OR)**

b) (i) Expand $f(z) = \frac{z}{(z+1)(z+2)}$ in the regions valid in $(i)|z+1| < 1$ and $(ii)1 < |z| < 2$. (7)

(ii) Evaluate $\int_0^{\infty} \frac{x^2}{(x^2+1)(x^2+9)} dx$ by contour integration. (7)

25. a) (i) Find $L\{tsin3t\}$ and $L^{-1}\left\{\frac{s+5}{(s-2)(s+3)}\right\}$. (7)

(ii) Find $L^{-1}\left\{\frac{s}{(s+1)(s^2+1)}\right\}$ by convolution theorem. (7)

**(OR)**

b) (i) Find $L\{f(t)\}$ if $f(t) = \begin{cases} k, 0 \le t \le a, \\ -k, a \le t \le 2a \end{cases}$ is the square wave function with $f(t) = f(t+2a)$. (7)

(ii) Solve using Laplace transform method: (7)

$y'' + y' - 2y = 3cos3t - 11sin3t, y(0) = 0$ and $y'(0) = 6$.

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