KUMARAGURU
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B.E / B.TECH DEGREE EXAMINATIONS: DEC 2014

(Regulation 2009)

Second Semester

MAT103: ENGINEERING MATHEMATICS - II

(Common To CSE/IT/TXT/FT/BT)

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions

PART A (10 x 1 = 10 Marks)

1. The value of $\int_0^1\int_0^2 e^y\, dxdy =$

   a) 2(1- e) b) 2 (e - 1)

   c) 2e d) 1

2. Volume of ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is

   a) $\frac{4zabc}{3}$ b) $\frac{4abc}{3}$

   c) $\frac{4yabc}{3}$ d) $\frac{4\pi abc}{3}$

3. The magnitude and the direction of the greatest change of $u = xyz^2$ at (1,0,3) is

   a) 9 in the direction of x axis b) 9 in the direction of y axis

   c) 9 in the direction of z axis d) 0

4. If R is a region bounded by a closed Curve C then the area of R is

   a) $\int_c xdy+ydx$ b) $\int xdy-ydx$

   c) $\int_c xdx+ydy$ d) $\frac{1}{2}\int_c xdy-ydx$

5. The function $f(z)=x^2+iy^3$ satisfies Cauchy – Riemann equation at

   a) x = 0, y = 0 b) x = 1, y = 0

   c) x = 0, y = 1 d) x = 1, y = 1

6. The conjugate harmonic of $u = x^2 - y^2$ is

a) $2xy$ b) $-2xy$

c) $x^2 + y^2$ d) $x + iy$

7 The invariant point of the transformation $w = \frac{1}{z - 2i}$ is

a) 1 b) I

c) 2i d) 4

8 The image of $x = \frac{\pi}{2}$ under w = cos z is

a) The imaginary axis of the w – plane b) The real axis of the w – plane

c) The positive side imaginary axis of the w – plane only d) An ellipse

9. If f(z) is analytic and its derivative $f'(z)$ is continuous at all points inside and on a simple closed curve C then

a) $\int_c f(z)dz = 2\pi i$ b) $\int_c f(z)dz = 0$

c) $\int_c f(z)dz = 2\pi i \Sigma R$ d) $\int_c f(z)dz = \Sigma R$

10 If the Laurent's expansion about z = a contains no negative powers of (z-a) then z = a is a

a) pole of order 1 b) pole of order 2

c) removable singularity d) essential singularity

PART B (10 x 2 = 20 Marks)

11. Evaluate $\iint dx\,dy$ over the region bounded by x = 0, x = 2, y = 0, y = 2.

12. Evaluate $\int_0^4 \int_0^2 \int_0^1 (x + y + z)\,dz\,dy\,dx$

13. Find the directional derivatives of $\phi = x^2 yz + 4xz^2 + xyz$ at (1,2,3) in the direction of $2\vec{i} + \vec{j} - \vec{k}$.

14. Determine the constant 'a' so that the vector $\vec{v} = 3x\vec{i} + (x + y)\vec{j} - az\vec{k}$ is solenoidal.

15. If $f(z) = u + iv$ is analytic then prove that the family of curves given by $u(x, y) = c_1$ & $v(x, y) = c_2$ from an orthogonal system.

16. Prove that $x^2 - y^2 + e^{-2x} \cos 2y$ is harmonic.

17. Find the image of the circle $|z| = 2$ under the transformation $w = 3z$.

18. Find the critical point of the transformation $w^2 = (z-\alpha)(z-\beta)$

19. Evaluate $\int_c \frac{z}{(z-2)^3} dz, C\, is\, |z| = 2$.

20. Classify the singularity of $f(z) = \frac{\cot \pi z}{(z-a)^2}$.

PART C (5 x 14 = 70 Marks)

21. a) (i) Find the smaller area bounded by $y = 2 - x \;\&\; x^2 + y^2 = 4$ (7)

(ii) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$. (7)

(OR)

b) (i) Find the area of the region outside the inner circle $r = 2\cos\theta$ and inside the outer circle $r = 4\cos\theta$ by double integration. (7)

(ii) By changing to polar coordinates, evaluate $\iint \frac{xy}{\sqrt{x^2+y^2}} dx\, dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$ (7)

22. a) (i) Show that $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational and find ϕ such that $\vec{F} = \nabla\phi$. (7)

(ii) Evaluate $\iint_s \vec{F}.\hat{n}\, ds$ where $\vec{F} = (x + y^2)\vec{i} + (2x)\vec{j} + (2yz)\vec{k}$ and S is a region bounded by $2x + y + 2z = 6$ in the first octant. (7)

(OR)

b) Verify divergence theorem for $\overrightarrow{F} = (x^2 - yz) + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ taken over the rectangular parallelepiped 0 < x < a, 0 < y < b, 0 < z < c.

23. a) (i) Find the analytic function $f(z)$ whose real part is $e^x(x\cos y - y\sin y)$ given *f (0)* =1. (7)

(ii) If f(z) is analytic prove that (7)

$$\left[\frac{\partial}{\partial x}|f(z)|\right]2^2 + \left[\frac{\partial}{\partial y}|f(z)|\right]^2 = |f'(z)|^2$$

(OR)

b) (i) Given that $u = \dfrac{\sin 2x}{\cosh 2y - \cos 2x}$ find the analytic function whose real part is u. (7)

(ii) Prove that an analytic function with constant real part is constant. (7)

24. a) (i) Show that the transformation $w = \dfrac{1}{z}$ transforms, in general circles and straight lines into circles and straight lines. Point out the circles and straight lines that are transformed into straight lines and circles respectively. (7)

(ii) Find the image in the w – plane of the region of the z – plane bounded by the straight line x = 1, y = 1 and x + y = 1 under the transformation $w = z^2$. (7)

(OR)

b) (i) Discuss the transformation w = sin z. (7)

(ii) Find the bilinear transformation which maps the points $z = 0, -i, -1$ into $w = i, 1, 0$. (7)

25. a) (i) Evaluate $\int_c \dfrac{z}{(z-1)(z-2)^2}\, dz, C\ is\ |z-2| = \dfrac{1}{2}$. (7)

(ii) Evalute $\int_0^\infty \dfrac{x^2 - x + 2}{x^2 + 10x + 9} dx$ (7)

(OR)

b) (i) Expand the function $f(z) = \dfrac{z}{(z-1)(z-3)}$ as Laurent series, valid in the region $3 < |z+2| < 5$ (7)

(ii) Evaluate $\int_0^{2\pi} \dfrac{\cos 3\theta}{5 + 4\cos\theta} d\theta$ (7)

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