

B.E / B.TECH DEGREE EXAMINATIONS: NOV/DEC 2014

(Regulation 2009)

Fifth semester

MAT108:NUMERICAL METHODS

(Common to ECE/IT)

Time: Three Hours

Maximum Marks: 100

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. A function $f(x)$ is continuous in the interval $(0,3)$. Then there is a root for $f(x)$ in the interval $(1,2)$ if
 - a) $f(1)$ and $f(2)$ are positive
 - b) $f(1)$ and $f(2)$ are negative
 - c) $f(1) > f(2)$
 - d) $f(1)$ and $f(2)$ are alternate in sign
2. The order of convergence for Newton Raphson method is
 - a) 1
 - b) 2
 - c) 3
 - d) 4
3. The relationship between Δ and ∇ is
 - a) $(1 + \Delta)(1 - \nabla) = 1$
 - b) $(1 + \nabla)(1 - \Delta) = 1$
 - c) $(1 - \nabla)(1 - \Delta) = 1$
 - d) $(1 + \nabla)(1 + \Delta) = 1$
4. The third order divided difference of a polynomial $x^3 + 3x^2 + 4x + 5$ is
 - a) 1
 - b) 4
 - c) 3
 - d) 5
5. The formula for first derivative at $x = x_0$ in Newton forward interpolation formula is
 - a) $\frac{1}{h} \left[\Delta y_0 - \frac{1}{2} \Delta^2 y_0 + \frac{1}{3} \Delta^3 y_0 - \frac{1}{4} \Delta^4 y_0 + \dots \right]$
 - b) $\left[\Delta y_0 - \frac{1}{2} \Delta^2 y_0 + \frac{1}{3} \Delta^3 y_0 - \frac{1}{4} \Delta^4 y_0 + \dots \right]$
 - c) $\frac{1}{h} \left[\nabla y_0 - \frac{1}{2} \Delta^2 y_0 + \frac{1}{3} \nabla^3 y_0 - \frac{1}{4} \Delta^4 y_0 + \dots \right]$
 - d) $\left[\Delta y_0 - \frac{1}{2} \nabla^2 y_0 + \frac{1}{3} \Delta^3 y_0 - \frac{1}{4} \nabla^4 y_0 + \dots \right]$
6. The truncation error for trapezoidal rule is

- a) $-\frac{1}{2}(x_n - x_0)h^2 f''(\eta)$ where $x_0 < \eta < x_n$
- b) $-\frac{1}{12}(x_n - x_0)h^2 f'''(\eta)$ where $x_0 < \eta < x_n$
- c) $-\frac{1}{12}(x_n - x_0)h^2 f''(\eta)$ where $x_0 < \eta < x_n$
- d) $-\frac{1}{2}(x_n - x_0)h^3 f''(\eta)$ where $x_0 < \eta < x_n$

7. _____ is a multi step method to find the solution of ODE

- a) Taylor series method b) Euler's method
c) Runge kutta method d) Milne's method

8. A particular case of Runge kutta method of second order is

- a) Taylor series method b) Euler's method
c) Improved Euler's method d) Milne's method

9. The order of convergence for Crank Nicolson method is

- a) 1 b) 2
c) 3 d) 4

10. To solve a Laplace equation we use the method

- a) Liebmann iteration process b) Euler's method
c) Crank Nicolson method d) Bender smith method

PART B (10 x 2 = 20 Marks)

11. Using Newton's method, find the root between 0 and 1 of $x^3=6x-4$
12. What are the advantages of iterative methods over direct methods for solving a system of linear equations?
13. Form the divided difference table for the data (0,1) (1,6) (3,40) and (4,80)
14. State Newton's forward interpolation formula.
15. Write down the expression for $\frac{d^2y}{dx^2}$ at $x = x_n$ by Newton's backward difference formula.
16. State Simpson's one- third rule.
17. Find $y(1.1)$ by solving the equation $\frac{dy}{dx} = x^2 + y^2$ $y(0) = 1$ using Taylor series
18. Write down the truncation error term for Runge kutta method of 4th Order
19. State whether the Crank Nicholson's scheme is an explicit or implicit scheme. Justify.
20. State Standard Five Point formula with relevant diagram.

PART C (5 x 14 = 70 Marks)

21. a) (i) Find the root of the equation $e^x = 2x + 1$, between 1 and 2, correct to four decimal places using Newton Raphson method (7)
 (ii) Solve the following system of Equation by using Gauss Elimination method (7)
 $5x_1 - x_2 = 9$, $-x_1 + 5x_2 - x_3 = 4$, $-x_2 + 5x_3 = -6$

(OR)

- b) (i) Solve the following system of equation by Gauss – Seidel’s iteration method (7)
 $10x_1 + 2x_2 + x_3 = 9$, $x_1 + 10x_2 - x_3 = -22$, $-2x_1 + 3x_2 + 10x_3 = 22$
 (ii) Find A^{-1} if $A = \begin{pmatrix} 8 & -4 & 0 \\ -4 & 8 & -4 \\ 0 & -4 & 8 \end{pmatrix}$ by Gauss Jordan Method (7)

22. a) Find $e^{-0.75}$ and $e^{-2.25}$ from the following data using both Newton forward and backward interpolation formula

x	1.00	1.25	1.50	1.75	2.00
e^{-x}	0.3679	0.2865	0.2231	0.1738	0.1353

(OR)

- b) (i) Use Stirling’s formula to compute $\tan 89^\circ 26'$ given the following table of values of $\tan x$ (7)

x	89°21	89°23	89°25	89°27	89°29
$\tan x$	88.14	92.91	98.22	104.17	110.90

- (ii) Apply Lagrange’s Interpolation formula to find $f(5)$ if $f(1)=2$, $f(2)= 4$, $f(3)= 8$, $f(4)=16$ and $f(7)=128$ (7)

23. a) Find the value of $f'(8)$, $f''(8)$, $f'(9)$ and $f''(9)$ from the following data using divided difference method

x	4	5	7	10	11
f(x)	48	100	294	900	1210

(OR)

- b) (i) Compute the value of π from the formula $\frac{\pi}{4} = \int_0^1 \frac{dx}{1+x^2}$ using trapezoidal rule with 10 sub intervals. (7)

- (ii) Evaluate $\int_0^1 \int_0^1 \frac{dx dy}{x+y+1}$ by using trapezoidal rule. (7)

24. a) A. Find the value of $y(0.2)$, $y(0.4)$ using Runge Kutta method of fourth order with $h = 0.2$ given that $\frac{dy}{dx} = \sqrt{x^2 + y}$ $y(0) = 0.8$

(OR)

- b) Given that $\frac{dy}{dx} = \frac{1}{2}(1 + x^2)y^2$, $y(0) = 1$, $y(0.1) = 1.06$, $y(0.2) = 1.12$ and $y(0.3) = 1.21$, Evaluate $y(0.4)$ and $y(0.5)$ by using Milne's Predictor corrector method.

25. a) Given that $\frac{\partial^2 f}{\partial x^2} - \frac{\partial f}{\partial t} = 0$, $f(0, t) = f(5, t) = 0$; $f(x, 0) = x^2(25 - x^2)$,

find the value of f up to 3 seconds taking the step size for x as $h = 1$

(OR)

- b) Given that $u(x, y)$ satisfies the equation $\nabla^2 u = 0$ and the boundary conditions $u(x, 0) = 0$, $u(x, 4) = 8 + 2x$, $u(0, y) = \frac{y^2}{2}$ and $u(4, y) = y^2$, find the value of $u(i, j)$ $i = 1, 2, 3$ and $j = 1, 2, 3$ correct to two places of decimals by Liebmann Iteration process.
