Register Number:……………………..

B.E DEGREE EXAMINATIONS: NOV/DEC 2014

(Regulation 2013)

Third Semester

U13MAT304: PARTIAL DIFFERENTIAL EQUATIONS AND FOURIER ANALYSIS

(Common to CEE/MEC)

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. A solution which contains as many arbitrary constants as there are independent variables is called

   a) singular integral b) complete integral

   c) general integral d) particular integral

2. The complete integral of pq = 1 is ________

3. The half range sine series corresponding to f(x) = x^2 expressed in the interval (0,5) converges at x=5 to ______

4. The value of b_n in the Fourier series expansion of x sinx in $(-\pi, \pi)$ is

   a) 0 b) 1

   c) π d) 2π

5. In the one dimensional wave equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$, the value of a^2 is

   a) $\frac{m}{T}$ b) $\frac{T}{m}$

   c) $\frac{m}{T}\frac{\partial y}{\partial t}$ d) $\frac{m}{T}\frac{\partial^2 y}{\partial t^2}$

6. The partial differential equation of the transverse vibration of a string is ______

7. In two dimensional heat flow, the temperature along the normal to the xy plane is______

8. The two dimensional steady state heat flow equation in polar coordinates is

   a) $r^2 \frac{\partial^2 u}{\partial r^2} + r\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0$ b) $r \frac{\partial^2 u}{\partial r^2} + r^2\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0$

c) $r^2\frac{\partial^2 u}{\partial r^2}+r\frac{\partial u}{\partial r}+\theta\frac{\partial^2 u}{\partial \theta^2}=0$ d) $r^2\frac{\partial^2 u}{\partial r^2}-r\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial \theta^2}=0$

9. If F(s) = F[f(x)] then $F\left[\int_a^x f(x)\,dx\right]$ is equal to

a) $\frac{F(s)}{-is}$ b) $\frac{F(s)}{is}$

c) $\frac{F(s)}{s}$ d) $\frac{-F(s)}{s}$

10. The Fourier cosine Transform of e^{-ax}, a>0 is __________

PART B (10 x 2 = 20 Marks)

(Not more than 40 words)

11. Form a differential equation by eliminating the arbitrary constants a and b from the equation $(x-a)^2+(y-b)^2=z^2\cot^2\alpha$.
12. Find the singular integral of the pde z = px + qy + p^2-q^2
13. Find the RMS value of y = x^2 in (- π,π).
14. What do you mean by Harmonic Analysis?
15. Classify the partial differential equation $4\frac{\partial^2 u}{\partial x^2}+4\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 u}{\partial y^2}-6\frac{\partial u}{\partial x}-8\frac{\partial u}{\partial y}-16u=0$.
16. A rod 30cm long has its ends ‘A’ & ‘B’ kept at $20^\circ c$ and $80^\circ c$ respectively until steady state conditions prevail. Find the steady state temperature in the rod.
17. Write any two solutions of the Laplace equation obtained by the method of separation of variables.
18. Write the boundary and initial conditions for the diameter of a semi circular plate of radius a is kept at 0^0C and the temperature at the semi circular boundary is T^0C.
19. Prove that $F[f(x-a)]=e^{ias}F[s]$.
20. Find Fourier Sine transform of $\frac{1}{x}$.

PART C (5 x 14 = 70 Marks)

(Not more than 400 words)

Q. No. 21 is compulsory

21. a) (i) Solve $(3z-4y)p+(4x-2z)q=(2y-3x)$ (7)

(ii) Solve $(D^3-7DD'^2-6D'^3)z=\sin(x+2y)+e^{2x+y}$ (7)

22. a) (i) Obtain the Fourier series for f(x) = 1 + x + x^2 in (-π, π). (7)

(ii) Find the half range cosine series $f(x) = \begin{cases} x \ \ in \ \ 0 \le x \le 1 \\ 2 - x \ \ in \ \ 1 \le x \le 2 \end{cases}$ (7)

(OR)

b) (i) Find the half range sine series f(x) = $l - x \ \ in \ \ 0 < x < l$ .Deduce $\sum_{n=1}^{\infty} \frac{1}{n^2}$ (7)

(ii) A function y=f(x) is given by the following table of values. Make the harmonic analysis of the function in (0, T) for the first harmonic. (7)

	0	T/6	T/3	T/2	2T/3	5T/6	T
y	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

23. a) A tightly stretched string with fixed end points x = 0 & x = ℓ is initially at rest in its equilibrium position. If it is set vibrating by giving each point a velocity $kx(\ell - x)$. Find the displacement of the string at any time.

(OR)

b) Two ends A and B of a rod ‘[illegible]’ cm long have the temp $40^\circ c \ \ and \ \ 90^\circ c$ until steady state prevails. The temperature at A and B are suddenly lowered to 0^0c .Find the temp distribution in the rod at time‘t’.

24. a) An infinitely long rectangular plate with insulated surface is 10 cm wide, and the two long edges and one short edges are kept at $0^\circ c$ temperature, while the other short edge x = 0 is kept at temperature given by

$u = \begin{cases} 20y & for \ 0 \le y \le 5 \\ 20(10 - y) & for \ 5 \le y \le 10 \end{cases}$. Find the steady state temperature distribution in the plate.

(OR)

b) The bounding diameter of a semi circular plate of radius ‘a’ cm is kept at 0^0C and the temperature along the semi circular boundary is given by

$$u(a,\theta)=\begin{cases}50\theta\ , when\ 0<\theta<\frac{\pi}{2}\\ 50(\pi-\theta)\ \ when\ \frac{\pi}{2}<\theta<\pi\end{cases}$$ Find the steady state temperature function

u (r , θ)

25. a) Find the Fourier transform of f(x) given by $f(x)=\begin{cases}1-x^2 & for\ \ |x|\le 1\\ 0 & for\ \ |x|\ge 1\end{cases}$ Hence

prove that i) $\int_0^\infty \left[\frac{\sin s - s\cos s}{s^3}\right]\cos\left(\frac{s}{2}\right)ds=\frac{3\pi}{16}$.

ii) $\int \left(\frac{\sin s - s\cos s}{s^3}\right)^2 ds=\frac{\pi}{15}$

(OR)

b) (i) Find the Fourier cosine transform of $e^{-a^2x^2}$ and hence find the Fourier sine transform of $xe^{-a^2x^2}$. (8)

(ii) Using transform method, evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$ (6)

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