KUMARAGURU
college of technology
character is life

Register Number:…………

B.E DEGREE EXAMINATIONS: NOV/DEC 2014

(Regulation 2013)

Third Semester

U13MAT304: PARTIAL DIFFERENTIAL EQUATIONS & FOURIER ANALYSIS

(Common to Mech/Civil)

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. The particular integral of the equation $(D^2 - 4D'^2)z = \sin(2x+y)$ is

a) $-\frac{x}{4}\sin(2x+y)$ b) $-\frac{1}{8}\cos(2x+y)$

c) $-\frac{x}{4}\cos(2x+y)$ d) $-\frac{x^2}{4}\cos(2x+y)$

2. A solution of partial differential equation which contains as many arbitrary functions as the order of the equation is called ________________ of the equation.

3. If $f(x) = x^2 + x,\ in(-2, 2)$, then the Fourier coefficient a_0 is

a) 0 b) $\frac{4}{3}$

c) $\frac{8}{3}$ d) 2

4. If $x = a$ is point of discontinuity of $f(x)$, then the Fourier series $f(x)$ converges at $x = a$ to ______________.

5. The suitable solution of one dimensional heat equation in transient state is

a) $u(x,t) = (Ae^{px} + Be^{-px})e^{p^2\alpha^2 t}$ b) $u(x,t) = (A\cos px + B\sin px)e^{-p^2\alpha^2 t}$

c) $u(x,t) = Ax + B$ d) $u(x,t) = (A\cos px + B\sin px)e^{p^2\alpha^2 t}$

6. Classify:

The partial differential equation $(x+1)u_{xx}-2(x+2)u_{xy}+(x+3)u_{yy}=0$ is __________.

7. The Laplace equation in polar coordinates is

a) $\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}=\frac{\partial u}{\partial t}$

b) $\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}=0$

c) $\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial \theta^2}=0$

d) $\frac{\partial^2 u}{\partial r^2}+\frac{\partial^2 u}{\partial \theta^2}=0$

8. In two dimensional heat equation $\alpha^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)=\frac{\partial u}{\partial t}$ where $\alpha^2=\frac{k}{\rho c}$, then α^2 is called ____________.

9. The Fourier sine transform of e^{-3x} is

a) $\frac{s}{s^2+3}$

b) $\frac{3}{s^2+3}$

c) $\frac{3^2}{s^2+3^2}$

d) $\frac{s}{s^2+3^2}$

10. If $F\{f(x)\}=F(s)$, then $\int_{-\infty}^{\infty}|f(x)|^2\,dx=$______________.

PART B (10 x 2 = 20 Marks)

(Not more than 40 words)

11. Form the partial differential equation by eliminating the arbitrary constants *a* and *b* from $ax^2+by^2+z^2=1$.

12. Obtain the complete solution of $yp=2xy+\log q$.

13. Determine the root mean-square value of the function $f(x)=x^2,\ in(-l,\ l)$.

14. What do you mean by harmonics and harmonic analysis in Fourier series?

15. Write down the all possible solutions of one dimensional wave equation.

16. A rod *AB* of length 10 cm has the ends *A* and *B* kept at temperature 40°C and 100°C respectively, until the steady state is reached. Find the steady state temperature distribution in the rod at time *t*.

17. Given the boundary conditions on a square or rectangular plate, how will you

identify the proper solution?

18. Write down the all possible solutions of Laplace equation in polar co-ordinates.

19. State convolution theorem on Fourier transforms.

20. If $F\{f(x)\}=F(s)$, then prove that $F\{f(x)\cos ax\}=\frac{1}{2}[F(s+a)+F(s-a)]$

PART C (5 x 14 = 70 Marks)
(Not more than 400 words)

Q.No. 21 is Compulsory

21. i) Discover the Fourier transform of $f(x)$, if (9)

$f(x)=\begin{cases}1-|x|, & for\ |x|<1\\ 0, & for\ |x|>1\end{cases}$. Hence prove that $\int_0^\infty \frac{\sin^4 x}{x^4}dx=\frac{\pi}{3}$.

ii) Solve for $f(x)$, the integral equation (5)

$\int_0^\infty f(x)\cos sx\,dx=\begin{cases}1-s, & for\ 0<s<1\\ 0, & for\ s>1\end{cases}$.

22. a) i) Form the partial differential equation by eliminating the arbitrary functions from $z=f(x+iy)+g(x-iy)$. (5)

ii) Solve the partial differential equation $(D^2-3DD'+2D'^2)z=e^{2x+3y}+x^2y$. (9)

(OR)

b) i) Obtain the complete and singular solution of the differential equation $z=px+qy+\sqrt{p^2+q^2}$. (7)

ii) Find the general solution of the equation $(x-2z)p+(2z-y)q=y-x$. (7)

23. a) Expand the function $f(x)=x(2l-x)\quad in\ (0,2l)$ as Fourier series, and hence deduce the sum of $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-....$

(OR)

b) i) Write the Fourier series expansion of $f(x)=\pi^2-x^2\quad in\quad -\pi<x<\pi$. (7)

ii) Derive the half-range sine series of $f(x)=a\quad in\ (0,1)$. Deduce the sum of $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+...\infty$. (7)

24. a) A uniform string is stretched and fastened to two points l apart. Motion is started by displacing the string into the form of the curve $y = k\sin^3\left(\frac{\pi x}{l}\right)$ and then releasing it from this position at time $t = 0$. Determine the displacement of the point of the string at a distance x from one end at time t.

(OR)

b) The ends A and B of a rod 40cm long have their temperatures kept at 0°C and 80°C respectively, until steady state conditions prevail. The temperature of the end B is then suddenly reduced to 40°C and kept so, while that of the end A is kept at 0°C. Obtain the subsequent temperature distribution $u(x, t)$ in the rod.

25. a) A rectangular plate with insulated surfaces is 20cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature of the short edge $x = 0$ is given by

$$u = \begin{cases} 10y, & for\ 0 \le y \le 10 \\ 10(20-y), & for\ 10 \le y \le 20 \end{cases}$$

and the two long edges as well as the other short edge are kept at 0°C, find the steady state temperature distribution in the plate.

(OR)

b) Along the inner boundary of a plate in the form of a circular annulus of radii 2 cm and 4 cm, the temperature is maintained at $(12\cos 2\theta + 20\sin 2\theta)$ and along the outer boundary, it is maintained at $(33\cos 2\theta + 50\sin 2\theta)$. Determine the steady state temperature at any point (r, θ) of the annulus.

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