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M.E DEGREE EXAMINATIONS: JAN 2015

(Regulation 2014)

First Semester

ENERGY ENGINEERING

P14MAT104: Applied Mathematics for Energy Engineering

Time: Three Hours   **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. The error function y is defined as   [K_2]

   a) $\dfrac{2}{\pi}\displaystyle\int_0^y e^{-u^2}\,du$ b) $\dfrac{2}{\sqrt{\pi}}\displaystyle\int_0^\infty e^{-u^2}\,du$

   c) $\dfrac{2}{\sqrt{\pi}}\displaystyle\int_0^y e^{-u^2}\,du$ d) $\dfrac{2}{\pi}\displaystyle\int_0^y e^{-u^2}\,du$

2. Match list I with list II and select the correct answer using the codes given below: [K_1]

List I	List II
A. $F\big(f(x-a)\big)$	1. $\dfrac{1}{2}\big[F(s+a)+F(s-a)\big]$
B. $F[f(ax)]$	2. $e^{isa}F(s)$
C. $F(af(x))$	3. $aF(s)$
D. $F(f(x)\cos as)$	4. $\dfrac{1}{a}F(\dfrac{s}{a})$

   Codes:

	A	B	C	D
(a)	4	2	3	1
(b)	2	4	3	1
(c)	4	3	2	1
(d)	2	3	4	1

3. The steps involved in finding the extremal is   [K_3]

   1. Solve the differential equation

   2. Apply Euler's Formula Solve the differential equation

   3. Identify the functional

   4. Substitute the boundary conditions and eliminate the constant.

   a) 1-2-4-3   b) 3-2-1-4

   c) 3-1-2-4   d) 2-1-4-3

4. Assertion(A) : The Rayleigh Ritz method is applicable only for solving variational problems. [K₄]

   Reason (R): The boundary value problems are solved by Rayleigh Ritz method.
   a) both A and R are individually true but R is the correct explanation of A
   b) both A and R are individually true but R is not the correct explanation of A
   c) A is true but R is false
   d) A is false but R is true

5. Match list I with list II(transformation of the parameters) and select the correct answer using the codes given below: [K₂]

List I	List II
A. Cycloid $[a(t-\sin t), a(1-\cos t)]$	1. $z = a\cos w + ib\sin w$
B. Hypocycloid	2. $z = a(\cos^3 w + i\sin w)$
C. Ellipse	3. $z = a(w + i - ie^{-iw})$
D. Parabola	4. $z = a(w^2 + 2i\,w)$

   Codes:

	A	B	C	D
(a)	4	2	3	1
(b)	2	4	3	1
(c)	3	2	1	4
(d)	2	3	4	1

6. The point at which velocity is zero is called _____ [K₂]
   a) saddle point
   b) stagnation point
   c) critical point
   d) invariant point

7. One of the roots of $x^3 - 4x + 1 = 0$ lies between _____ [K₃]
   a) (0,1)
   b) (1,2)
   c) (2,3)
   d) (1/2,1)

8. Assertion (A) : In Gauss Jacobi method ,any error made in computation is not corrected in the subsequent iteration [K₂]

   Reason(R) : Iteration Method is self correcting method
   a) both A and R are individually true but R is the correct explanation of A
   b) both A and R are individually true but R is not the correct explanation of A
   c) A is true but R is false
   d) A is false but R is true

9. In solving poisson equation numerically, which of the following formulas are useful? [K₂]

   (i) Standard five point formula   (ii) Diagonal five point formula

   (iii) explicit scheme   (iv) Leibmann's formula

   a) (i) and (ii)
   b) (ii) and (iii)
   c) (i), (ii) and (iii)
   d) (i), (ii) and (iv)

10. The steps involved in finding the solution of 1-D heat equation is [K₃]

    1) Replace the derivatives by difference quotients

    2) Choose the spacing h , k and find λ

3) Select explicit or implicit scheme

4) Find the temperature at mesh points

a) 1-2-4-3   b) 1-2-3-4

c) 3-1-2-4   d) 2-1-4-3

PART B (10 x 2 = 20 Marks)

11. If u and $\dfrac{\partial u}{\partial x} \to 0$ as $x \to \infty,$ determine the Fourier cosine transform of $\dfrac{\partial^2 u}{\partial x^2}$.   [K$_3$]

12. Estimate the Fourier Sine transform of f(x) = 1 in $(0,l)$.   [K$_4$]

13. Test for an extremum of the functional $\int\limits_{x_0}^{x_1} \left(y^2 + 2\,xy\,y' \right) dx.$ [K$_4$]

14. Prove that the shortest distance between two points plane is a straight line.   [K$_2$]

15. Prove that for closed polygon, the sum of the exponents $\dfrac{\alpha_1}{\pi} - 1,\ \dfrac{\alpha_2}{\pi} - 1,\ ....,\dfrac{\alpha_n}{\pi} - 1$ in the   [K$_2$]

Schwarz-Christoffel transformation is equal to -2.

16. Compose the equation for the complex potential for the flow in the w - plane with constant   [K$_6$]

velocity v_0 under the transformation $w = z + \dfrac{a^2}{z}.$

17. The equation $x^2 + ax + b = 0$ has two real roots α and β. Then in the iteration method   [K$_4$]

$x_{k+1} = -\left(x_k^2 + b\right)/a$ is convergent near α. Detect the region of convergence.

18. Gauss-Seidel method is better than Gauss-Jacobi method. Justify.   [K$_2$]

19. Classify the equation $\left(1 + x^2\right)\dfrac{\partial^2 u}{\partial x^2} + \left(5 + 2x^2\right)\dfrac{\partial^2 u}{\partial x \partial t} + \left(4 + x^2\right)\dfrac{\partial^2 u}{\partial t^2} = 0.$ [K$_4$]

20. Identify the value of $\lambda,$ for which the explicit method of solving the hyperbolic equation   [K$_2$]

$\dfrac{\partial^2 u}{\partial x^2} = \dfrac{1}{c^2}\dfrac{\partial^2 u}{\partial t^2}$ is stable, where $\lambda = \dfrac{ck}{h}$?

PART C (6 x 5 = 30 Marks)

21. Determine the distribution of temperature in the semi-finite medium $x \geq 0$, when the end x = 0 is maintained at zero temperature and the initial temperature is $f(x)$ [K$_3$]

22. Find the extremals of the following functional $\int\limits_{0}^{\pi/2} [2xy + (\dfrac{dx}{df})^2 + (\dfrac{dy}{df})^2] df$ subject to   [K$_4$]

$x(0) = x(\pi/2) = -1,\ y(0) = 0,\ y(\pi/2) = 1.$

23. Assess the transformation that maps the semi – infinite strip of width π bounded by the lines $v=0, v=\pi$ and $u=0$ of the w plane into the upper half of the z plane. [K₄]

24. Apply Gauss – Seidel iterative method to find the values of x,y and z (Correct to three decimal places) $10x+y+z=12$, $2x+10y+z=13$, $x+y+5z=7$. [K₅]

25. Measure the temperature at each mesh points of the PDE $u_{xx}=u_t$ subject to $u(0,t)=0, u(4,t)=0, u(x,0)=x(4-x)$ assuming h = 1 up to t = 4 using Bender-Schmidt relation. [K₅]

26. Find the complex potential due to a source at $z=-a$ and a sink at $z=a$ of equal strength k. Also determine the equipotential lines and stream lines. [K₄]

PART D (4 x 10 = 40 Marks)

27. Using Fourier transform, find a function $u(x,y)$ which is harmonic in the open square $0<x<\pi, \ 0<y<\pi$ takes a constant value u_0 on the edge $y=\pi$ and vanishes on the other edges of the square. [K₄]

28. Solve the boundary value problem $y''+y+x=0$ given that $y(0)=y(1)=0$ using Ritz method. Also compare with exact solution. [K₅]

29. Determine a positive root of the equation $x^x=100$ by Newton –Raphson Method. [K₅]

30. Construct the values at the interior points of the Laplace equation $\nabla^2 u=0$ satisfying $u(0,y)=0$, $u(3,y)=3y+9$, $u(x,0)=x^2$ and $u(x,3)=2x^2$ by Leibmann's method , correct to two decimal places. [K₅]

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