KUMARAGURU college of technology
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M.E DEGREE EXAMINATIONS: JAN 2015

(Regulation 2014)

First Semester

EMBEDDED SYSTEMS

P14MAT106: Applied Mathematics for Embedded Systems

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. **Assertion(A):** The characteristic equation of one dimensional wave equation is $\xi(x,t)=x-ct \;\; and \;\; \eta(x,t)=x+ct$. [K2]

   Reason(R) : The one dimensional wave equation is $u_t = ku_{xx}$.

   a) A is wrong and R is correct
   b) A is correct and R is wrong
   c) Both A and R are individually correct and R implies A
   d) Both A and R are individually correct and A implies R

2. If $\bar{A}\, u_{\xi\xi} + \bar{B}\, u_{\xi\eta} + \bar{C}\, u_{\eta\eta} + \bar{D}\, u_{\xi} + \bar{E}\, u_{\eta} + \bar{F}u = \bar{G}$ be the canonical form of $Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu = G$ then $\bar{A}$ is [K1]

   a) $2A\xi_x\eta_x + B(\xi_x\eta_y + \xi_y\eta_x) + 2C\xi_y\eta_y$
   b) $A\,\xi_x{}^2 + B\xi_x\xi_y + C\xi_y{}^2$
   c) $A\,\eta_x{}^2 + B\eta_x\eta_y + C\eta_y{}^2$
   d) $A\xi_{xy} + B\xi_{xx} + D\xi_x + E\xi_y$

3. Match the following. [K1]

List I	List II
A) $\frac{d}{dx}\left[x^n J_n(x)\right] =$	1) 0
B) $\frac{d}{dx}\left[x^{-n} J_n(x)\right] =$	2) 1
C) $J_0 + 2J_2 + 2J_4 + 2J_6 + ... =$	3) $x^n\, J_{n-1}(x)$
D) $J_1(0) =$	4) $-x^{-n}\, J_{n+1}(x)$

Codes:

	A	B	C	D
a)	2	3	1	2
b)	3	4	1	2
c)	3	4	2	1
d)	1	3	2	4

4. The steps involved in obtain the series solution when $x = 0$ is an ordinary point of [K2]

the equation $P_0(x)\frac{d^2y}{dx^2}+P_1(x)\frac{dy}{dx}+P_2(x)y=0$.

1. Substituting the values of $a_2, a_3, a_4, \ldots$ in the solution.

2. Calculate $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$ and substituting these values in the differential equation.

3. Assumed solution of the equation is $y=\sum_{r=0}^{\infty} a_r x^r$.

4. Equate the coefficient of various powers of x to zero and determine $a_2, a_3, a_4, \ldots$ in terms of a_0, a_1.

a) 1 – 3 – 4 – 2 b) 1 – 2 – 3 – 4
c) 3 – 2 – 4 – 1 d) 2 – 3 – 1 – 4

5. If $f(x)=|x|$ is expanded as a Fourier series is $(-\pi,\pi)$ then a_o is [K3]

a) 2π b) $-\pi$
c) π^2 d) π

6. Z – Transform of $(-1)^n$ is [K3]

a) $\frac{z}{z-1}$ b) $\frac{z}{z+1}$
c) $\frac{z^n}{z^n+1}$ d) $\frac{z^n}{(z+1)^n}$

7. **Assertion(A):** The mean and variance of the moment generating function $\left(\frac{2}{3}+\frac{1}{3}e^t\right)^9$ are 3 and 2 [K2]

Reason(R) : The moment generating function of binomial distribution is $(q+pe^t)^n$ and its mean, variance are *np*, *npq* respectively.

a) A is correct and R is not correct b) A is wrong and R is correct
c) Both A and R are wrong d) Both A and R are correct

8. If a random variable X has the following probability distribution [K3]

Values of X = x	0	1	2	3	4	5	6	7	8
$p(x)$	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

Then the value of a is

a) $\frac{1}{81}$ b) 81
c) 1 d) $\frac{1}{10}$

9. The inter arrival time in a queueing system follows a ______________. [K1]

a) Poisson Distribution b) Exponential distribution
c) Uniform distribution d) Binomial distribution

10. Match the following: [K2]

In the Kendall's notation $(a / b / c) : (d / e)$

List I		List II	
A	'*e*' stands for	1	Service distribution
B	'*b*' stands for	2	Arrival distribution
C	'*a*' stands for	3	Number of service channels
D	'*c*' stands for	4	Queue discipline

Codes:

	A	B	C	D
a)	2	3	1	4
b)	3	1	4	2
c)	4	1	2	3
d)	3	4	1	2

PART B (10 x 2 = 20 Marks)

11. If u(x, t) is a function of x and t, prove that $L\left[\frac{\partial^2 u}{\partial t^2}\right] = s^2U(x,s) - s\,u(x,0) - u_t(x,0)$. [K3]

12. Write the D'Alembert's solution of wave equation. [K2]

13. Classify the singular points of the equation $x^3(x-2)y'' + x^2y' + 6y = 0$. [K3]

14. Write the generating function of Legendre polynomials. Hence find $P_n(-1)$. [K4]

15. Obtain the Discrete Fourier Transform {*x* (n)}, given that *x* (n) = a^n, for *n* = 0, 1, 2, … N – 1 and *x* (*n* + N) = *x* (n). [K4]

16. Compute $Z\left(\frac{1}{n}\right)$. [K3]

17. A random variable (continuous) X has a probability density function $f(x) = 3x^2, 0 \le x \le 1$. find '*a*' such that $P(x \le a) = P(x > a)$. [K3]

18. State any two properties of normal distribution. [K2]

19. Write Little's formula. [K1]

20. People arrive at a Theatre ticket booth in Poisson distributed arrival rate of 25 per hour. Service time is constant at 2 minutes. Calculate the mean waiting time in the waiting line. [K3]

PART C (6 x 5 = 30 Marks)

21. Reduce the following equation to canonical form and hence solve it. [K3]
$y\,u_{xx} + (x+y)\,u_{xy} + x\,u_{tt} = 0.$

22. Show that $e^{\frac{x}{2}\left(t-\frac{1}{t}\right)}$ is a generating function of Bessel's function. [K3]

23. Obtain the Fourier cosine series of $f(x) = x$ in (0, 2). [K4]

24. Using convolution theorem, find the inverse Z – Transform of $\frac{z^2}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}$. [K3]

25. After correcting the proof up to 50 pages of a book, the proof reader found that there are three errors per 5 pages. Use Poisson distribution and estimate the number of pages with 0, 1, 2, 3 error and more than 3 errors in a book of 1000 pages. [K_3]

26. Customers arrive at a one – man barber shop according to a Poisson process with a mean inter – arrival time of 12 minutes. The customers spend an average of 10 minutes in the chair [K_5]

  (a) What is the expected number of customers in the shop and in the queue.
  (b) What percentage of time can arrivals walk straight into the barber's chair without having to wait.
  (c) How much time can a customer expect to spend in the shop?
  (d) Find the average time spent in the queue.
  (e) Find the probability that there are more than 3 customers in the system.

PART D (4 x 10 = 40 Marks)

27. Using Laplace Transform technique, solve initial and boundary value problem described as PDE $u_{xx} = \frac{1}{c^2}u_{tt} - cos\omega t;\ 0 < x < \infty.$ [K_5]

B.Cs: $u(0, t) = 0$, $u(x, t)$ is bounded as $x \to \infty$.

I.Cs: $u(x, 0) = u_t(x, t) = 0$.

28. State and prove the Orthogonal property of Legendre's polynomial. [K_4]

29. Compute the moment generating function of geometric distribution and hence its mean and variance. [K_4]

30. An automatic car wash facility operates with only one bay. Cars arrive according to a Poisson distribution with a mean of 4 cars / hr. and may wait in the facility's parking lot if the bay is busy. Find L_s, L_q, W_s, W_q, if the service time. [K_5]

  i) is constant and equal to 10 minutes.
  ii) follows uniform distribution between 8 and 12 minutes.
  iii) follows normal distribution with mean 12 minutes and S.D. 3 minutes.
  iv) follows a discrete distribution with values 4, 8 and 15 minutes with corresponding probabilities 0.2, 0.6 and 0.2.

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