Register Number: ………………..

B.E DEGREE EXAMINATIONS: MAY 2015

(Regulation 2009)

Sixth Semester

AERONAUTICAL ENGINEERING

AER127:Theory of Elasticity

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. An Euler-Bernoulli beam in bending is assumed to satisfy
   - a) Both plane stress as well as plane strain conditions
   - b) Plane strain condition but not plane stress condition
   - c) Plane stress condition but not plane strain condition
   - d) Neither plane strain condition nor plane stress condition
2. In three-dimensional linear elastic solids, the number of non-trivial stress-strain relations, strain-displacement equations and equations of equilibrium are respectively
   - a) 3, 3 & 3
   - b) 6, 3 & 3
   - c) 6, 6 & 3
   - d) 6, 3 & 6
3. A body undergoes deformation under plane strain conditions when subjected to the following stresses (in MPa): $\sigma_{xx} = 450$, $\sigma_{yy} = 450$, $\tau_{xy} = 75$, $\tau_{xz} = 0$, $\tau_{yz} = 0$. What are the remaining components of stresses (in MPa) and strains? Assume the material to be isotropic and linear-elastic with Young's modulus E = 200 GPa and poisson's ratio □ = 1/3.
   - a) $\sigma_{zz} = 0$, $\varepsilon_{xx} = 0.00225$, $\varepsilon_{yy} = 0.00225$, $\gamma_{xy} = 0.002$, $\gamma_{xz} = 0$
   - b) $\sigma_{zz} = 300$, $\varepsilon_{xx} = 0.001$, $\varepsilon_{yy} = 0.001$, $\gamma_{xy} = 0.001$, $\gamma_{xz} = 0$, $\gamma_{yz} = 0$
   - c) $\sigma_{zz} = 300$, $\varepsilon_{xx} = 0.00225$, $\varepsilon_{yy} = 0.00225$, $\gamma_{xy} = 0.001$, $\gamma_{xz} = 0$, $\gamma_{yz} = 0$
   - d) $\sigma_{zz} = 0$, $\varepsilon_{xx} = 0.001$, $\varepsilon_{yy} = 0.001$, $\gamma_{xy} = 0.002$, $\gamma_{xz} = 0$, $\gamma_{yz} = 0$
4. Following stress state is proposed for a two- dimensional problem with no body forces: $\sigma_{xx} = 3x^2y + 4y^2$, $\sigma_{yy} = y^3 + 14xy$, $\tau_{xy} = -3xy^2 - 7x^2$. It satisfies
   - a) Equilibrium equations but not compatibility equation
   - b) Compatibility equations but not equilibrium equation
   - c) Neither equilibrium equations nor compatibility equation
   - d) Both equilibrium equations and compatibility equation

5. Which of the following Airy's stress functions could satisfy the given boundary conditions, assuming constant values of σ_{xx} = P, σ_{yy} = Q, & τ_{xy} = R along the boundary?

a) $\phi = P(x^2/2) + Q(y^2/2) - R_{xy}$
b) $\phi = P(y^2/2) + Q(x^2/2) + R_{xy}$
c) $\phi = P(y^2/2) + Q(x^2/2) - R_{xy}$
d) $\phi = P(x^2/2) + Q(y^2/2) + R_{xy}$

6. For a plane strain problem, the stresses satisfy the condition

a) $\tau_{xz} = \tau_{yz} = \sigma_z = 0$
b) $\tau_{xz} = \tau_{yz} = 0, \sigma_z = \nu(\sigma_x + \sigma_y)$
c) $\tau_{xz} = \tau_{yz} = 0, \sigma_z = \nu\tau_{xy}$
d) $\tau_{xz} = \tau_{yz} = 0, \sigma_z = \nu(\sigma_x + \sigma_y) + (1-\nu)\tau_{xy}$

7. In the absence body moments the symmetry of the stress tensor is derived from

a) Force equilibrium conditions
b) Moment equilibrium conditions
c) Linear relations between stresses & strains
d) Compatibility conditions

8. The compatibility conditions in theory of elasticity ensure that

a) there is compatibility between various direct and shear stresses
b) relationships between stresses and strains are consistent with constitutive relations
c) displacements are single valued and continuous
d) stresses satisfy bi-harmonic equation

9. The tubes will not warp under torsion are called

a) Single cell tubes
b) Multi – cell tubes
c) Pressurized tubes
d) Neuber tubes

10. According to the Prandtl's stress function $\phi = \phi(x, y)$, which one of the following is true

a) $\tau_{xz} = \frac{\partial \phi}{\partial x}; \tau_{yz} = -\frac{\partial \phi}{\partial y}$
b) $\tau_{xz} = \frac{\partial \phi}{\partial y}; \tau_{yz} = -\frac{\partial \phi}{\partial x}$
c) $\tau_{xz} = -\frac{\partial \phi}{\partial x}; \tau_{yz} = \frac{\partial \phi}{\partial y}$
d) $\tau_{xz} = -\frac{\partial \phi}{\partial y}; \tau_{yz} = \frac{\partial \phi}{\partial x}$

PART B (10 x 2 = 20 Marks)

11. What are the differences between mechanics of materials approach and theory of elasticity approach?
12. Define Section modulus.
13. State St.Venant's principle.
14. The following stress distribution has been determined at a point. Do these stresses satisfy equilibrium in the absence of body forces?
15. Differentiate plane stress and plane strain problems.
16. What are axisymmetric problems? Give example.

17. For the stress function, $\phi = Ar^2 \log r$ (where A is constant), compute the stress components σ_r and σ_θ.

18. Determine the maximum stress of a wedge carrying concentrated load at its vertex with a semi vertex angle is 30 degree.

19. What is Navier's theory of torsion?

20. A bar of equilateral angle of side 200 mm is subjected to a torque of 100N-m.Estimate the angle of twist per unit length? Assume, G = 60 KN/mm^2.

PART C (5 x 14 = 70 Marks)

21. a) The stress tensor is given by $\begin{bmatrix} 20 & 15 & 12.5 \\ 15 & 5 & 75 \\ 12.5 & 75 & 10 \end{bmatrix}$. Determine the stress invariants and the principal stresses?

(OR)

b) Using the 3-D stress –strain relations, Prove that $E\nabla^2 u + \frac{\partial \theta}{\partial x} + 3X = 0$, $E\nabla^2 v + \frac{\partial \theta}{\partial y} + 3Y = 0$, $E\nabla^2 w + \frac{\partial \theta}{\partial z} + 3Z = 0$

22. a) Derive the compatibility equations for the plane strain condition in Cartesian coordinates.

(OR)

b) (i) What is Stress ellipsoid? List out the significance of ellipsoid with an example. (7)

(ii) Derive the relationship among E, G and K by theory of elasticity approach. (7)

23. a) A simply supported beam of length 2L is subjected to uniformly distributed load of intensity 'q' over the entire length of the beam. Obtain the expression for bending stress and compare the result with strength of materials solution.

(OR)

b) Show how the Bi-harmonic equation ($\nabla^4 \phi = 0$) in 2 –D cartesian coordinate system can be transformed into that polar coordinates using Airy's stress function.

24. a) Determine the stress concentration in an infinite stressed plate with a circular hole subjected to uni–axial tension.

(OR)

b) Obtain the stresses in a thick cylinder subjected to internal and external pressures. Sketch the variation of stresses through the thickness.

25. a) Determine the maximum and minimum shearing stresses and angle of twist per unit length for a shaft of triangular cross sections.

(OR)

b) (i) State Prandtl's theory of torsion and prove that $T = 2\iint \phi \, dx \, dy$. (7)

(ii) Show that the polar moment of inertia of a non – circular section is $J = \iint_A \left[x^2 + y^2 + x\frac{\partial \psi}{\partial y} - y\frac{\partial \psi}{\partial x} \right] dxdy$ using St.Venant's warping function. (7)

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