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Register Number: ………………..

B.E / B.TECH DEGREE EXAMINATIONS: JUNE 2015

(Regulation 2009)

First Semester

MAT101: ENGINEERING MATHEMATICS I

(Common to all Branches)

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. The eigenvalues of $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 3 \end{bmatrix}$

   a) 1,4,3 b) 3,4,0
   c) 1,2,3 d) 0,0,3

2. If A is a singular matrix at least one of the eigenvalues of A is

   a) infinite b) 1
   c) -1 d) zero

3. The angle of POQ, if P(2,3,-6) and Q(3,-4,5) is

   a) $Cos^{-1}(-18/35)$ b) $Cos^{-1}(18\sqrt{2}/35)$
   c) $Cos^{-1}(-36/35)$ d) $Cos^{-1}(-18\sqrt{2}/35)$

4. The centre and radius of the sphere $x^2+y^2+z^2-4x-6y-8z+4=0$ is

   a) (2,3,4) & 5 b) (2,3,4) & 25
   c) (4,3,2) & 25 d) (4,3,2) & 5

5. The first derivative of xy=12 at (c,c) is

   a) 2/c b) 1/c
   c) -1 d) 2

6. The envelope of the family of lines $y = mx + am^2$, m is a parameter is

   a) $y^2+4ax = 0$ b) $x^2+4ay = 0$
   c) $y^2-4ax = 0$ d) $x^2-4ay = 0$

7. If u and v are functions of x and y then if $J=\frac{\partial(u,v)}{\partial(x,y)}$ *and* $J'=\frac{\partial(x,y)}{\partial(u,v)}$ then JJ' is

a) -1	b) 1
c) 2	d) 0

8. The stationary point of $x^3y^2(a-x-y)$ is

a) (a/2,a/3)	b) (a/2,a/2)
c) (a/2,1/3)	d) (a/3,a/3)

9. The particular solution of $(D^2 + 2D+1)y = e^{2x}$ is

a) $e^{-2x}/13$	b) $e^{2x}/9$
c) $e^{2x}/12$	d) $e^{2x}/10$

10. The complimentary function of $(D^2-4)y = x^2$ is

a) $A+Be^{-2x}$	b) $Ae^{-x} + Be^{2x}$
c) $Ae^{-2x} + Be^{2x}$	d) $(Ax+B)e^{2x}$

PART B (10 x 2 = 20 Marks)

11. State Cayley Hamilton theorem.
12. If the eigenvalues of A are 2,3,4 then find the eigenvalues of Adj A.
13. Obtain the equation to the line which passes through (1,4,-1) and has direction ratios 2,-1,4.
14. Find the direction ratios of the line joining the points P(4,2,-5) and Q(-2,0,-8).
15. What is the radius of curvature of y = ax+b?
16. List any two properties of evolute.
17. If $u=\frac{x^2}{y}$, $v=\frac{y^2}{x}$ find $\frac{\partial(x,y)}{\partial(u,v)}$
18. State the condition for the functions $u=f_1(x,y,z)$, $v=f_2(x,y,z)$ and $w= f_3(x,y,z)$ to be functionally dependent.
19. Solve $(D^2-3D+2)y=0$.
20. Transform $(1+x^2)y''+(1+x)y'+y=4\cos\log(1+x)$ into a differential equation with constant coefficients.

PART C (5 x 14 = 70 Marks)

21. a) i) Verify Cayley Hamilton theorem of the matrix $A=\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}$ (5)

ii) Diagonalise the matrix $A=\begin{bmatrix}1&1&3\\1&5&1\\3&1&1\end{bmatrix}$ by orthogonal Transformation. (9)

(OR)

b) i) Obtain the eigen values and eigen vectors of $A=\begin{bmatrix}1&1&3\\1&5&1\\3&1&1\end{bmatrix}$. (7)

ii) If x_1 and x_2 are two eigenvectors corresponding to two different eigenvalues λ_1 and λ_2 of a real symmetric matrix A then prove that x_1 and x_2 are orthogonal. (7)

22. a) i) Prove that lines $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ and $\frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}$ intersect. (7)

ii) Show that the spheres $x^2+y^2+z^2+7x-11y+5z-4=0$ and $x^2+y^2+z^2+4x+2y+6z+22=0$ cut each other orthogonally. (7)

(OR)

b) i) Find the equation of the plane through the point (1,0,-2) and perpendicular to the planes 2x+y-z=2 and x-y-z=3. (7)

ii) Show that the plane 4x+9y+14z=64 touches the sphere $3(x^2+y^2+z^2)$ -2x-3y-4z =22 and find the point of contact. (7)

23. a) i) Find the equation of the circle of curvature of xy=12 at the point (3,4). (7)

ii) Find the envelope of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where a and b are connected br the relation $a^2+b^2 = c^2$, c being a constant. (7)

(OR)

b) i) Find the radius of curvature of the curve at the point θ $x=a\log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$, $y = a \sec \theta$. (7)

ii) Find the envolute of the $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. (7)

24. a) i) If $x=r\sin\theta\cos\phi,\quad y=r\sin\theta\sin\phi,\quad z=r\cos\theta$ find $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$. (7)

ii) Find the extreme value of $x^2+y^2+z^2$ where x + y +z =3a. (7)

(OR)

b) i) Using Lagrange's multipliers method find the extreme values of $a^3x^2 + b^3y^2+c^3z^2$ where xy +yz+zx=xyz (7)

ii) Prove that of all rectangular parallelepipeds of a given volume cube has the least surface area. (7)

25. a) i) Solve $(D^2+2D+2)y= 2e^{-x}\sin x$ (7)

ii) Solve $(x^2D^2+xD-1)y=x^2\log x$ (7)

(OR)

b) i) Solve $(D^2+4)\, y = \tan 4x$ using variation of parameters method. (7)

ii) Solve $dx/dt - 7x + y = 0$; $dy/dt - 2x - 5y = 0$. (7)

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