KUMARAGURU
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Register Number: ………………..

B.TECH DEGREE EXAMINATIONS: JUNE 2015

(Regulation 2009)

Second Semester

MAT103: ENGINEERING MATHEMATICS II

(Common to CSE/FT/BIO/IT/TXT)

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. The area of the upper half of the circle $x^2 + y^2 = 1$ is

a) $\frac{\pi}{2}$ b) π

c) 4π d) 2π

2. Transform the integral $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\, dy\, dx$ into polar co-ordinates

a) $\int_0^{\pi}\int_0^{a} r^2\, dr\, d\theta$ b) $\int_0^{\pi/2}\int_0^{a} r^2\, dr\, d\theta$

c) $\int_0^{\pi/2}\int_0^{a/2} r^2\, dr\, d\theta$ d) $\int_0^{\pi}\int_0^{a} dr\, d\theta$

3. If the vector $\vec{F} = (x-3y)\vec{i} + (y-2z)\vec{j} + (x+kz)\vec{k}$ is Solenoidal, then the value of 'k' is

a) -2 b) 4

c) 2 d) 1

4. If S is any closed surface enclosing a volume V and $2x\vec{i} + 2y\vec{j} + 3z\vec{k}$, then $\int_S \vec{F}.\hat{n}\, ds$ is

a) 7πv/3 b) 7v

c) 7πv d) 7πv /2

5. Cauchy – Riemann equations for the function f(z) = u (x, y) + i v(x, y) is

a) $u_x = v_y\, ;\, u_y = -v_x$ b) $u_x = u_y\, ,\, v_x = -v_y$

c) $u_y = v_x\, ,\, u_x = -v_y$ d) $u_x = v_y\, ,\, u_y = v_x$

6. The function $f(z)=\frac{1}{z^2+1}$ is analytic everywhere except

a) $z=1$ b) $z=\pm 1$

c) $z=\pm i$ d) $z=i$

7. A point is called a critical point of the transformation if at that point

a) f' (z) ≠ 0 b) f' (z) = 0

c) f (z) ≠0 d) f (z) = 0

8. The invariant point of the transformation $w=\frac{1}{z-2i}$ is

a) 1 b) i

c) 2i d) 4

9. The Cauchy's integral formula for the nth derivative of an analytic function f(z) is

a) $f^n(a)=\frac{n!}{2\pi i}\int_c \frac{f(z)}{(z-a)^{n+1}}dz$ b) $f^n(a)=\frac{(n-1)!}{2\pi i}\int_c \frac{f'(z)}{(z-a)^{n}}dz$

c) $f^n(a)=\frac{n!}{2\pi i}\int_c \frac{f(z)}{(z-a)^{n}}dz$ d) $f^n(a)=\frac{n!}{2\pi i}\int_c \frac{f(z)}{(z-a)^{n-1}}dz$

10. The value of $\int_c \frac{dz}{2z+1}$ where C is $|z|=1$

a) 0 b) $2\pi i$

c) πi d) $-\pi i$

PART B (10 x 2 = 20 Marks)

11. Evaluate $\int_0^1\int_0^x\int_0^{x+y} dxdydz$

12. Find the area of the circle r =a cosθ using double integration

13. In what direction from (3,1,-2) is the directional derivative of $\phi=x^2y^2z^4$ maximum?

14. Using Green's theorem find the area of the ellipse

15. Prove that $e^x\cos y$ is a harmonic function

16. At what point the function $f(z)=x^2+iy^2$ is analytic

17. Define conformal mapping

18. Under the transformation $w=\frac{1}{z}$ find the image $|z-2i|=2$

19. Evaluate $\int_C \frac{dz}{(z-3)^2}$ where C is $|Z|=1$

20. Find the singularities of $f(z) = \dfrac{\cot \pi z}{(z-a)^3}$

PART C (5 x 14 = 70 Marks)

21. a) (i) Change the order of integration and evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy dx$ (8)

(ii) Find the area of the cardioid r = a (1+ cosθ) using double integration (6)

(OR)

b) (i) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ (8)

(ii) Transform into polar co-ordinates and then evaluate $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2 + y^2) dx\, dy$ (6)

22. a) (i) Prove that $\nabla^2 r^n = n(n+1) r^{n-2}$ and hence deduce that $\nabla^2 (1/r) = 0$ (7)

(ii) Prove $F = (2xy + z^3) i + x^2 j + 3xz^2 k$ is a conservative force. Find φ so that $\nabla\phi = F$ (7)

(OR)

b) Verify Gauss divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ taken over the rectangular parallelepiped $0 \le x \le a \quad 0 \le y \le b \quad 0 \le z \le c$

23. a) Find the analytic function of f(z) and find its conjugate harmonic

$u = \dfrac{\sin 2x}{\cosh 2y - \cos 2x}$

(OR)

b) (i) If w =f(z) is an analytic function then prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$ (8)

(ii) If f(z) =u+iv is an analytic function then prove that $u(x,y) = C_1$ and $v(x,y) = C_2$ cut each other orthogonally where C_1 and C_2 are constants (6)

24. a) Plot the image under the mapping $w = z^2$ of the triangular region bounded by y=1 ,x=1 and x+y=1

(OR)

b) (i) Show that the map $w = \frac{1}{z}$ maps the totality of circles and lines as circles or lines (7)

(ii) Find the bilinear transformation which maps the points z = 0,1,∞ into w = i,1,-i (7)

25. a) Using contour integration evaluate $\int_{0}^{2\pi} \frac{d\theta}{13+5\sin\theta}$

(OR)

b) i Obtain the Laurent's series expansion $f(z)=\frac{z-1}{(z+2)(z+3)}$ *for* $2<|z|<3$ (6)

ii Evaluate $\int_{C} \frac{z+4}{z^2+2z+5}dz$ where C is $|z+1+i|=2$ (8)

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