Register Number: ………………..

B.E DEGREE EXAMINATIONS: MAY 2015

(Regulation 2009)

Fourth Semester

ELECTRONICS AND COMUNICATION ENGINEERING

MAT107:Random Processes and Vector Spaces

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. The distribution function of a random variable X is $F(x)=1-(1+x)e^{-x};x\succ 0$. Then its density function is

   a) e^{-x} b) x e^{-x}

   c) 1 + x d) $x^2\ e^{-x}$

2. If X is a uniformly distributed random variable in (0, 1). Then its mean is

   a) 2 b) 0

   c) ½ d) ¼

3. If X and Y are two independent random variables, then Cov (X, Y) is

   a) 0 b) x^2+y^2

   c) $E(xy)=E(x)+E(y)$ d) $E(xy)=E(x)/E(y)$

4. The coefficient of correlation is independent of

   a) Change of scale b) Change of origin

   c) Neither (a) nor (b) d) Both (a) and (b)

5. If the process X (t) satisfies the condition $\underset{T\to\infty}{Lt}\ V\ (\overline{X_T})=0$, then it is known as

   a) mean ergodic b) correlation ergodic

   c) markov d) poisson

6. If in a markov chain, for every i, j and for some n such that $p_{ij}^{(n)}\succ 0$, then it is called as

   a) return state b) periodic state

   c) irreducible d) non-null persistent

7. The spectral density function of a real random process is an

   a) onto function b) odd function

c) one-to-one function d) even function

8. If the input of a time-invariant stable linear system is a WSS, then the output function is

a) not defined b) also a WSS process

c) ergodic process d) markov process

9. Le t S be a set of vectors and it is linearly independent, then the subset of S is

a) linearly independent b) linearly dependent

c) mutually independent d) non linearly dependent

10. If $v_1 = (1,2,3),\ then\ \|v_1\|$ is

a) 14 b) 4

c) 7 d) $\sqrt{14}$

PART B (10 x 2 = 20 Marks)

11. A continuous random variable that can assume any value between x = 2 and x =5 has the density function $f\,(x) = k\ (1 + x)$. Find k.

12. State the properties of MGF.

13. The joint pdf of (X, Y) is given by $e^{-(x+y)}$; $0 \prec x, y \prec \infty$. Are X and Y independent?

14. If $Y = -2X + 3,\ \ find\ the\ \ Cov\ (X, Y).$

15. Classify the random process.

16. If the customers arrive at a bank according to Poisson process with mean rate of 2 per minute. Find the probability that during one minute interval no customers arrive.

17. Calculate the mean and variance of a stationary random process whose auto correlation function is given by $R_{XX}\,(\tau) = 18 + \dfrac{2}{6 + \tau^2}$.

18. Define the properties of power spectral density spectrum.

19. H be the set of vectors of the form {a-3b, b-a, a, b}, where a and b are scalars. Show that H is a subspace of R^4.

20. Define inner product space.

PART C (5 x 14 = 70 Marks)

21. a) (i) The diameter of an electric cable, say X is assumed to be a continuous random variable with $f\ (x) = 6x(1-x),\ 0 \le x \le 1.$ (7)

i). Check that f (x) is a pdf.

ii). Determine a and b such that $P(x \prec b) = P(x \succ b)$.

iii). Find $P\left(x \le \frac{1}{2} \Big/ \frac{1}{3} \prec x \prec \frac{2}{3} \right)$

(ii) Six dice are thrown 729 times. How many times do you expect at least three dice to show a 5 or 6? (7)

(OR)

b) (i) The daily consumption of milk in a city in excess of 20,000 liters is approximately exponentially distributed. The average excess in consumption of milk is 3000 liters. The city has a daily stock of 35,000 liters. What is the probability that, of two days selected at random, the stock is insufficient for both the days. (7)

(ii) The mean yield for one acre plot is 662 kgs with S.D 32. Assuming normal distribution, how many one acre plots in a batch of 1000 plots would you expect to yield. (7)

i). over 700 kgs. ii). below 650 kgs. iii). What is the lowest yield of the best 100 plots?

22. a) (i) From a sack of fruits containing 3 oranges, 2 apples and 3 bananas, a random sample of 4 pieces of fruit is selected. If X is the number of oranges and Y is the number of apples in the sample, find (7)

i). the joint probability distributions of x and y

ii). $P(X + Y \le 2)$.

(ii) The joint pdf of the two dimensional random variable (x,y) is given by (7)

$$f(x, y) = \begin{cases} \frac{8xy}{9}, & 1 \le x \le y \le 2 \\ 0 & , otherwise \end{cases}$$

. Find the marginal and conditional density functions of x and y.

(OR)

b) (i) Let X and Y have the p.m.f $P(x_1, x_2) = \frac{x_1 + 2x_2}{18}$, $x_1 = 1, 2;\ x_2 = 1, 2.$ Find the $Cov(X_1, X_2)$. (4)

(ii) Let the p.d.f be $f(x, y) = \frac{x + y}{3}$, $0 \le x \le 1;\ 0 \le y \le 2.$ Calculate the regression line of X on Y. (10)

23. a) (i) Discuss the stationarity of the random process $X(t) = A\cos(\omega_0 t + \theta)$ *if A and* ω_0 *are constants and* θ *is a uniformaly distributed random variable in* $(0, 2\pi)$. (7)

(ii) A man either drives a car or catches a train to go to office each day. He never goes 2 days in row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first (7) day of the week, the man tossed a fair dice and drove to work if and only if a 6 appeared. Find i). the probability that he takes a train on the third day

ii). the probability that he drives to work in the long run.

(OR)

b) (i) If the WSS process is given by $X(t) = 10\cos(100t + \theta)$, *where* θ *is uniformly distributed over* $(-\pi, \pi)$. Prove that $X(t)$ *is correlation.ergodic.* (7)

(ii) Prove that the sum of two independent Poisson processes is a Poisson process but not the difference of the two. (7)

24. a) (i) State the properties of cross correlation function. (4)

(ii) Find the power spectral density of the random process, if its auto correlation function is given by $R_{XX}(\tau) = e^{-\alpha|\tau|} \, Cos\, \beta\tau.$ (10)

(OR)

b) (i) Consider a white Gaussian noise of zero mean and power spectral density $\frac{N_0}{2}$ applied to a low pass RC filter whose transfer function is $H(f) = \frac{1}{1 + i\,2\pi f RC}$. Find the auto correlation function of the output random process. Also find the mean square value of the output process. (7)

(ii) If the output of a time-invariant stable linear system is a WSS process, then show that the output is also a WSS process. (7)

25. a) (i) Let W be the subspace of R^5 spanned by $u_1 = (1,2,-1,3,4),\ u_2 = (2,4,-2,6,8),\ u_3 = (1,3,2,2,6), u_4 = (1,4,5,1,8),$ $u_5 = (2,7,3,3,9)$. *Find the subset of the vectors that form a basis of* W. 7

(ii) v_1 *and* v_2 *are the vectors in the inner product space, prove that*

$$\|v_1 + v_2\|^2 + \|v_1 - v_2\|^2 = 2\,\|v_1\|^2 + 2\,\|v_2\|^2.$$ (7)

(OR)

b) (i) What is the role of Gram-Schmidt algorithm? (2)

(ii) Find an orthonormal basis for the subspace W of R^3, generated by (3,0,4), (-1, 0, 7) and (2,9,11) using Gram-Schmidt algorithm. (12)

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