



6. The number of vertices and edges in  $K_5$  are
 

a) 5, 10	b) 5, 15
c) 5, 20	d) 5, 25
7. In an incidence matrix A, a row with all 0's represents \_\_\_\_\_
 

a) a pendent vertex	b) a vertex of degree two
c) an isolated vertex	d) a vertex of degree three
8. The definition of \_\_\_\_\_ matrix makes no provision for parallel edges.
 

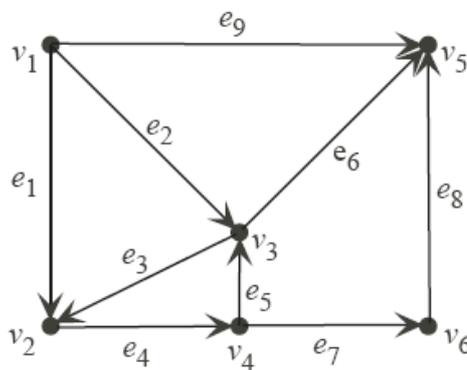
a) Path	b) circuit
c) incidence	d) adjacency
9. The rank of a connected graph with 5 vertices is \_\_\_\_\_
 

a) 5	b) 1
c) 4	d) 0
10. The method used for finding shortest spanning tree is \_\_\_\_\_
 

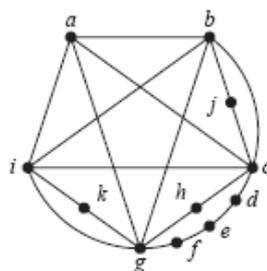
a) Kruskal algorithm	b) Simplex algorithm
c) Hungarian algorithm	d) Min-cut algorithm

**PART B (10 x 2 = 20 Marks)**

11. Distinguish between walk and path.
12. Find the strongly connected components for the following digraph.

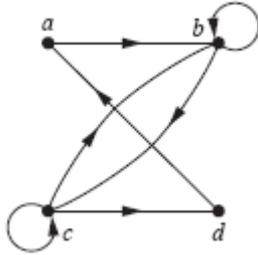


13. Draw a graph with diameter 4 and radius 2.
14. Define internal vertex.
15. Give an example for bi-chromatic graph.
16. Is the given graph G is planar?



17. Define path matrix of a graph.

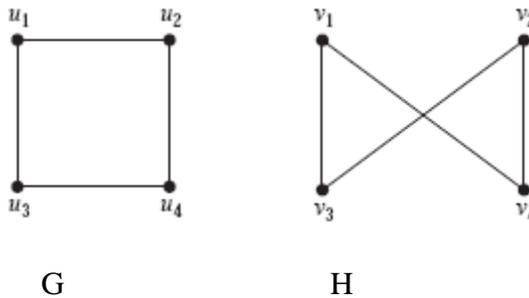
18. Determine the adjacency matrix of the given directed graph with respect to the vertices listed in alphabetic order.



19. Define minimal spanning tree.  
20. State Max-Flow Min-cut theorem.

**PART C (5 x 14 = 70 Marks)**

21. a) (i) Show that the following graphs  $G = (V, E)$  and  $H = (W, F)$  are isomorphic. (7)



- (ii) Prove that a simple graph with  $n$  vertices and  $k$  components can have at most  $\frac{(n-k)(n-k+1)}{2}$  edges. (7)

**(OR)**

- b) (i) If  $G$  is a simple graph with  $n$  vertices with  $n \geq 3$  such that the degree of every vertex in  $G$  is at least  $\frac{n}{2}$ , then prove that  $G$  has a Hamilton circuit. (8)  
(ii) Explain different types of digraphs with example. (6)

22. a) (i) Prove that any connected graph with  $n$  vertices and  $n - 1$  edges is a tree. (7)  
(ii) Show that every tree has either one or two centers. (7)

**(OR)**

- b) (i) What is the ordered rooted tree that represents the expression  $((x + y)^2 + (x - 4)/3)$ ? (7)  
(ii) Show that a graph  $G$  with  $n$  vertices,  $n - 1$  edges and no circuits is connected. (7)

23. a) (i) Prove that every tree with two or more vertices is 2- chromatic. (7)  
(ii) Show that a graph with at least one edge is 2-chromatic if and only if it has no circuits of odd length. (7)

(OR)

b) If a planar embedding of a connected graph  $G$  has  $n$  vertices,  $m$  edges and  $f$  regions, then prove that  $f + n = m + 2$ .

24. a) (i) If  $A(G)$  is an incidence matrix of a connected graph  $G$  with  $n$  vertices then (7)  
prove that the rank of  $A(G)$  is  $n - 1$

(ii) If  $B$  is a circuit matrix of a connected graph  $G$  with  $e$  edges and  $n$  vertices then (7)  
Show that rank of  $B$  is  $e - n + 1$ .

(OR)

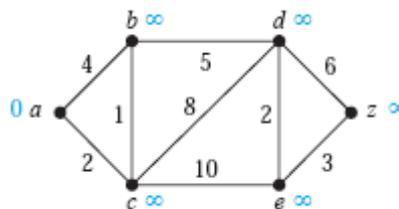
b) Explain the following with example:

(i) Incidence matrix

(ii) Circuit matrix

(iii) Adjacency matrix

25. a) Use Prim's algorithm to find the length of a shortest path between the vertices  $a$  and  $z$  in the weighted graph  $G$ .



(OR)

b) (i) Prove that every connected graph has at least one spanning tree. (7)

(ii) Show that a flow  $f$  of a network  $N$  is maximum if and only if there are no  $f$ -improvable paths in  $N$ . (7)

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