KUMARAGURU
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Register Number: ……………………..

B.E DEGREE EXAMINATIONS: APRIL 2015

(Regulation 2009)

Seventh Semester

COMPUTER SCIENCE ENGINEERING

MAT112: Partial Differential Equations and Their Solution Methodologies

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. Solution of the equation $\frac{\partial z}{\partial y} = 0$ is

   a) x = 0 a) $z = x f(y) + \emptyset(y)$

   c) z = f(x) c) $z = \emptyset(y)$

2. The particular integral of the equation $(D^2 + DD')z = e^y$ is

   a) x e^y b) e^y

   c) 2x e^y d) y e^y

3. The canonical form of the wave equation $u_{tt} = 4u_{xx}$ when $\xi = x - 2t, \eta = x + 2t$ is

   a) $u_{\eta\eta} = 0$ b) $u_{\xi\eta} = 4$

   c) $u_{\xi\xi} = 0$ d) $u_{\xi\eta} = 0$

4. The type of partial differential equation $u_{xx} - u_{yy} = 0$ is

   a) Parabolic b) hyperbolic

   c) elliptic d) Both hyperbolic and elliptic

5. The partial differential equation that represents 2D steady state heat flow in Cartesian coordinates is

   a) $u_{xx} - u_{yy} = 0$ b) $u_{xx} + u_{yy} = 0$

   c) $u_x - u_y = 0$ d) $u_x + u_y = 0$

6. If $u(r,\theta) = (A\log r + B)(C\theta + D)$ is the solution of Laplace equation in polar coordinates then it is called

a) Circular harmonics of degree two.
b) Circular harmonics of degree 'n'.
c) Circular harmonics of degree zero
d) Circular harmonics of degree one

7. If $\int_0^t N(t)dt = 0$, for all t>0 then N(t) is called

a) Unit function
b) Heaviside function
c) Dirac delta function
d) Null function

8. $L\left(\frac{\partial y}{\partial x}\right) =$ ______

a) $\frac{dy}{dx}$
b) $\frac{d\bar{y}}{dx}$
c) $\frac{\partial y}{\partial x}$
d) $\frac{\partial \bar{y}}{\partial x}$

9. $F\left[e^{-a|x|}\right], a < 0$ is

a) does not exist
b) $\sqrt{\frac{2}{\pi}}\frac{s}{s^2+a^2}$
c) $\sqrt{\frac{2}{\pi}}\frac{a}{s^2+a^2}$
d) $\sqrt{\frac{2}{\pi}}\frac{a}{s^2-a^2}$

10. $F_s^{-1}\left(\frac{1}{s}\right) =$ ______

a) does not exist
b) 1
c) ∞
d) 0

PART B (10 x 2 = 20 Marks)

11. Form the PDE by eliminating the arbitrary constants a and b from $z = axe^y + \left(\frac{1}{2}\right)a^2e^{2y} + b$.

12. Identify the particular integral of $\left(D^2 + 3DD' + 2D'^2\right)z = x + y$.

13. Classify the equation $xyr - \left(x^2 - y^2\right)s - xyt + py - qx = 2\left(x^2 - y^2\right)$.

14. What is the necessity to transform PDE to canonical form?

15. Explain the method of separation of variables.

16. Give three possible solutions of the equation $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$.

17. Find the Laplace transform of $\frac{\partial^2 u}{\partial t^2}$ where u is a function of two variables x and t.

18. Determine the inverse Laplace transform of $\frac{1}{\sqrt{s}}$.

19. Evaluate $F_s\left(\frac{\partial^2 u}{\partial x^2}\right)$.

20. Find the Fourier Cosine transform of f(x) =k in $(0, l)$.

PART C (5 x 14 = 70 Marks)

21. a) (i) Form the PDE by eliminating the arbitrary functions for $z = \frac{1}{x}\varphi(y-x) + \varphi'(y-x)$. (7)

(ii) Find the singular integral of $z = px + qy + \log(pq)$. (7)

(OR)

b) (i) Solve: $(x+2z)p + (4zx - y)q = 2x^2 + y$. (7)

(ii) Solve: $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y\cos x$. (7)

22. a) Reduce the equation $(y-1)u_{xx} - (y^2-1)u_{xy} + y(y-1)u_{yy} + u_x - u_y = 2ye^{2x}(1-y)^3$ to a canonical form and solve it.

(OR)

b) Reduce the equation $u_{xx} - 2\sin x\, u_{xy} - \cos^2 x u_{yy} - \cos x\, u_y = 0$ to a canonical form and solve it.

23. a) Find the steady state temperature distribution in a thin rectangular plate bounded by the lines $x = 0, x = a, y = 0, y = b$. The edges $x = 0, x = a, y = 0$ are kept at temperature zero while the edge $y = b$ is

kept at $100^\circ C$.

(OR)

b) A thin semi-circular plate of radius a has its boundary diameter kept at $0^\circ C$ and its circumference at $100^\circ C$. If $u(r,\theta)$ is the steady state temperature, find $u\left(\frac{a}{4},\frac{\pi}{2}\right)$.

24. a) Using the Laplace transform method, find the solution $\theta(x,t)$ of one dimensional diffusion equation $\frac{\partial^2\theta}{\partial x^2} = \frac{1}{k}\frac{\partial\theta}{\partial t}$, $0 \le x \le \pi, t > 0$ satisfying the boundary conditions $\theta(0,t) = 1 - e^{-t}, t > 0$

$$\theta(\pi,t) = 0,\ t > 0$$

$$\theta(x,0) = 0, t = 0, 0 \le x \le \pi$$

(OR)

b) A string is stretched and fixed between two points (0. 0)and (a, 0). Motion is initiated by displacing the string in the form $u = b\sin\left(\frac{\pi x}{a}\right)$ and released from rest at time t=0. Using Laplace transform method, find the displacement of any point on the string at any time t.

25. a) Using Transform method, solve the boundary value problem $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, u(0,t) = 1, u(\pi,t) = 3, u(x,0) = 1$ where $0 < x < \pi, t > 0$.

(OR)

b) Using Transform method, determine a function $u(x,y)$ which is harmonic in the open square $0 < x < \pi, 0 < y < \pi$, takes a constant value $u_\circ$ on the edge $y = \pi$ and vanishes on other edges of the square.

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