Register Number:……..

KUMARAGURU
college of technology
character is life

M.E DEGREE EXAMINATIONS: JUNE 2015

(Regulation 2014)

Second Semester

COMPUTER SCIENCE AND ENGINEERING

P14MATE11: Stochastic Models and Simulation

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. Assertion (A) : Probability of impossible event is zero [K_3]
Reason (R): Events are mutually independent

a) R is the correct reason for A b) A is not correct

c) R is not correct d) R is not the correct reason for A

2. A box contains 6 red and 4 white balls. If 3 balls are drawn at random, the probability of getting 2 white balls without replacement, is [K_3]

a) 1/20 b) 18/125

c) 4/25 d) 3/10

3. Arrange the following in the appropriate sequence: [K_3]

A: Poisson Variable B: Binomial Variable C: Bernoulli variable D: Normal variable

a) C,B, A, D b) A,B, C, D

c) A,D, B C d) D, C, A, B

4. Match the appropriate item from the RHS with those on LHS. [K_2]

List I	List II
A.MGF of Poisson distribution	i. $\frac{e^{bt}-e^{at}}{(b-a)t}$
B. MGF of Binomial distribution	ii. $e^{\lambda(e^t-1)}$
C. MGF of Exponential distribution	iii. $(q+pe^t)^n$
D. MGF of Uniform distribution	iv. $\frac{\lambda}{\lambda-t}$

a) A - ii, B – iii, C – iv, D - i b) A - iii, B – ii, C – iv, D - i

c) A - ii, B – iii, C – i, D - iv d) A - ii, B – iv, C – iii, D - i

5. The tpm of a Markov chain is a stochastic matrix, then the sum of all elements of any row is equal to [K_2]

a) 0 b) 1

c) -1 d) 2

6. If a Markov chain is finite and irreducible, then all its states are [K_1]

a) Transient b) Steady state

c) Persistent d) Non-null persistent

7. (i). The Poisson process is a Markov process [K_4]

(ii). The inter arrival time of a Poisson process is an exponential distribution.

a) Both (i) and (ii) are wrong. b) (i) is true and (ii) is wrong

c) (i) is wrong and (ii) is true d) Both (i) and (ii) are true.

8. Messages arrive at a telegraph office in accordance with the laws of a Poisson process with a mean rate of 3 messages per hour. What is the probability that no message will arrived during the morning hours between 8 a.m and 12 noon? [K_4]

a) e^{12} b) e^{-12}

c) e^{8} d) e^{-8}

9. Explain the term "Trafic intensity" [K_1]

a) $\frac{mean\ service\ rate}{mean\ arrival\ rate}$ b) Mean arrival rate

c) Mean service rate d) $\frac{mean\ arrival\ rate}{mean\ service\ rate}$

10. If the mean arrival rate is 1/12 and the mean service rate is ¼ in a (M/M/1):(∞ /FIFO), then the expected number of customers in the system is [K_4]

a) 1 b) 2

c) 1/2 d) 1/3

PART B (10 x 2 = 20 Marks)

11. A card is drawn at random from a well shuffled deck of 52 cards. Find the probability of drawing a king or a queen. [K_2]

12. A continuous random variable X has pdf $f(x) = 3x^2, 0 \leq x \leq 1,$ Find a such that [K_3]

$P(X \leq a) = P(X > a)$.

13. If the sum of mean and variance of a Binomial distribution is 4.8 for 5trials. Find the distribution. [K_4]

14. If X is uniformly distributed over (0, 10), calculate the probability that X > 6. [K_4]

15. Define irreducible Markov chain? [K_1]

16. If the tpm of the Markov chain is $\begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix}$, find the steady state distribution of the chain. [K_4]

17. Show that the Poisson process is a Markov process [K_4]

18. State Chapman-Kolmogorov theorem. [K_1]

19. Define Little's formula. [K_1]

20. Discus the term Balking and Reneging. [K_4]

PART C (6 x 5 = 30 Marks)

21. A problem is given to 3 students X, Y, Z whose chances of solving it are ½, 1/3, 2/5 respectively. What is the probability that the problem is solved? [K_3]

22. Let the random variable X has the pdf $f(x) = \begin{cases} \frac{1}{2}e^{-x/2}, x > 0 \\ 0, \ otherwise \end{cases}$. Find the mgf and mean of X. [K_4]

23. In a book of 520 pages, 390 typographical errors occur. Assuming Poisson's law for the number of errors per page, find the probability that a random sample of 5 pages will contain no error. [K_3]

24. Consider a Markov chain with 2 states and tpm $P = \begin{pmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{pmatrix}$. Find the stationary probabilities of the chain [K_3]

25. Prove that the sum of two independent Poisson processes is again a Poisson process [K_2]

26. Customers arrive at the first class ticket counter of a theatre at a rate of 12 per hour. There is one clerk servicing the customers at the rate of 30 per hour. [K_1]

    (i) What is the probability that there is no customer at the counter?

(ii) What is the probability that there are more than 2 customers at the counter?

PART D (4 x 10 = 40 Marks)

27. Find the mgf of Binomial distribution and hence find its mean and variance [K_4]

28. [K_2]

The tpm of a Markov chain with three states 0, 1, 2 is $P = \begin{pmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{pmatrix}$ and the initial state distribution of the chain is $P(X_0 = i) = \frac{1}{3}, i = 0, 1, 2$. Find (i) $P(X_2 = 2)$ and (ii) $P(X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2)$

29. A random variable x has the following probability function . [K_4]

x	0	1	2	3	4	5	6	7
p(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

(i) Find k

(ii) Evaluate P(X < 6) and $P(X \geq 6)$

(iii)Find the minimum value of c such that $P(X \leq c) > \frac{1}{2}$.

(iv)Evaluate $P(1.5 < X < 4.5/X > 2)$.

(v) Find P(X < 2) and P(X > 3).

30. A super market has two girls attending to sales at the counters. If the service time for each customer is exponential with mean 4 min. and if people arrive in poisson fashion at the rate of 10 per hour. 10 [K_3]

(i) What is the probability that a customer has to wait for service?

(ii) What is the expected percentage of ideal time for eaCHJ GIRL?

(iii)If the customer has to wait in the queue, what is the expected length of his waiting time?

****