KUMARAGURU
college of technology
character is life

B.E / B.TECH DEGREE EXAMINATIONS: JUNE 2015

(Regulation 2013)

First Semester

U13MAT101: ENGINEERING MATHEMATICS-I

(Common to all Branches)

Time: Three Hours   **Maximum Marks: 100**

Answer all the Questions

PART A (10 x 1 = 10 Marks)

1. Given $A = \begin{bmatrix} a & h & g \\ 0 & b & f \\ 0 & 0 & c \end{bmatrix}$ then select the suitable options

    a) The characteristic equation is
        $|A - \lambda I| = 0$
    b) The Eigen values are a,b,c

    c) The characteristic equation is
        $(\lambda-a)(\lambda-b)(\lambda-c) = 1$
    d) The Eigen values are h,g,f

2. The rank of a zero matrix is _____

3. Envelope of the family of lines $y = mx + \frac{a}{m}$ is

    a) $y^2 = ax$
    b) $x^2 = ay$
    c) $x^2 = 4ay$
    d) $y^2 = 4ax$

4. The radius of curvature of the straight line is _____

5. The Integrating factor of $\frac{dy}{dx} + \frac{y}{x} = x^3$ is

    a) x
    b) e^x
    c) log x
    d) $\frac{x^4}{e^4}$

6. The differential equation $p = \log(px - y)$ has the solution _____

7. The complementary solution of the equation is $(D^2 - 2D + 1) y = 0$

    a) $(Ax + B)e^x$
    b) $y = (Ax - B)e^x$
    c) $y = (Ax - B)e^{-x}$
    d) $(Ax + B)e^{-x}$

8. The wronskian value of the two solutions f(x)=sin2x and g(x)=cos2x is _____

9. $\dfrac{\partial(u,v)}{\partial(x,y)} \times \dfrac{\partial(x,y)}{\partial(u,v)} =$

a) 0

b) 1

c) -1

d) 2

10. If $AC - B^2 < 0$, at the stationary point (x_1, y_1) then the stationary point is called a _____

PART B (10 x 2 = 20 Marks)

(Not more than 40 words)

11. Two eigen values of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ are equal to 1 and 5. Find the eigen values of A^{-1}.

12. State Cayley – Hamilton theorem.

13. The radius of curvature and centre of curvature at a point P on a curve are 5 and $(1,2)$ what is the circle of curvature at P?

14. Find the envelope of $x\cos\alpha + y\sin\alpha = 1$ where α is the parameter.

15. Solve $p^2 - 6p + 8 = 0$ where $p = \dfrac{dy}{dx}$.

16. Solve $y = px + \sin^{-1}(p)$.

17. Solve $(D^2 + 4)y = 0$.

18. Convert the differential equation $x^2 y'' - xy' + y = \log x$ into a differential equation with constant coefficients.

19. If $x^y + y^x = a^K$ find $\dfrac{dy}{dx}$.

20. If $u = 2xy, v = x^2 - y^2, x = r\cos\theta, y = r\sin\theta, find \dfrac{\partial(u,v)}{\partial(r,\theta)}$.

PART C (5 x 14 = 70 Marks)

(Not more than 400 words)

Q.No. 21 is Compulsory

21.   i)   Find the general solution of $y = xp + \sqrt{p^2 + 1}$ (7)

     ii)   Solve $(x+1)\dfrac{dy}{dx} + 1 = e^{-y}$.   (7)

22. a) i) Find the eigen values and eigen vectors of $A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$ (7)

   ii) Using Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ find A^{-1}. (7)

(OR)

b) Reduce the quadratic form given below into canonical form through an orthogonal transformation and also find the rank, signature, index and nature of the quadratic form $2x_1^2 + x_2^2 + x_3^2 + 2x_1x_2 - 2x_1x_3 - 4x_2x_3$

23. a) i) Prove that the radius of the curvature at any point θ on the curve (7)

   $x = a(\theta - \sin\theta); \ y = a(1 - \cos)$ is $4a\sin\dfrac{\theta}{2}$.

   ii) Find the envelope of the family of straight lines $\dfrac{x}{a} + \dfrac{y}{b} = 1$ where a and b are (7)

   connected by the relation $a^2 + b^2 = c^2$.

(OR)

b) i) Find the evolute of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. (7)

   ii) Find the envelope of the family of lines $\dfrac{x}{a}\cos\theta + \dfrac{y}{b}\sin\theta = 1, \theta$ being the (7)

   parameter.

24. a) i) Solve $\dfrac{d^2 y}{dx^2} + 4\dfrac{dy}{dx} + 4y = e^{-x}\sin 2x$ (7)

   ii) Solve $x^2\dfrac{d^2 y}{dx^2} + x\dfrac{dy}{dx} + y = 4\sin(\log x)$ (7)

(OR)

b) i) Solve $y'' + 4y = \tan 2x$ by the method of variation of parameters. (7)

   ii) Solve the simultaneous equations $\dfrac{dx}{dt} + y = e^t \ and \ x - \dfrac{dy}{dt} = t$ (7)

25. a) i) Find the maximum and minimum values of $x^3 y^2 (1 - x - y)$. (7)

   ii) A rectangular box open at the top is to have a volume of 32cc. Find the (7) dimensions of the box, that requires the least material for its construction.

(OR)

   b) i) Expand $e^x \sin y$ in powers of x and y up to the third degree terms. (7)

   ii) Show that the function $u = xy + yz + zx, v = x^2 + y^2 + z^2$ & $w = x + y + z$ (7) are functionally dependent and find the relationship between them.

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