KUMARAGURU college of technology — character is life

Register Number:…………..….

B.E DEGREE EXAMINATIONS: MAY 2015

(Regulation 2013)

Third Semester

ELECTRICAL AND ELECTRONICS ENGINEERING

U13MAT309**:** Partial Differential Equations and Transforms

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. The order of the PDE, while eliminating two arbitrary functions, is

   a) 1 b) 2

   c) 0 d) More than 2

2. The complete solution of $z = px + qy$ is……………..

3. The number of boundary conditions are required to solve completely $\frac{\partial u}{\partial t}=a^2\frac{\partial^2 u}{\partial x^2}$ is

   a) 2 b) 4

   c) 3 d) 1

4. The nature of the equation $4\frac{\partial^2 u}{\partial x^2}+4\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 u}{\partial y^2}-6\frac{\partial u}{\partial x}-8\frac{\partial u}{\partial y}-16u=0.$ is……………..

5. In the Fourier expansion of $f(x)=\begin{cases}1+\frac{2x}{\pi}, & -\pi<x<0\\ 1-\frac{2x}{\pi}, & 0<x<\pi\end{cases}$ in $(-\pi,\pi)$, the value of b_n is

   a) 4 b) $\frac{4}{n^2}$

   c) 0 d) $\frac{4}{n\pi^2}$

6. If $f(x)=\begin{cases}\cos x & if\ 0<x<\pi,\\ 50, & if\ \pi<x<2\pi,\end{cases}$ and $f(x)=f(x+2\pi)$ for all x, the sum of the Fourier series of $f(x)\, at\, x=\pi$ is……………..

7. $F\left[e^{-a|x|}\right],\ a<0$ is

   a) does not exist b) $\sqrt{\frac{2}{\pi}}\frac{s}{s^2+a^2}$

c) $\sqrt{\frac{2}{\pi}}\frac{a}{s^2+a^2}$ d) $\sqrt{\frac{2}{\pi}}\frac{a}{s^2-a^2}$

8. If $F_s(s)$ is the Fourier sine transform of f(x) then $F_s[f(x)\cos ax]$ is ……………..

9. The value of $z\{f(n)\}$ when $f(n)=na^n$ is

a) $\frac{az}{(z-a)^2}$ b) $\frac{az}{(z+a)^2}$

c) $\frac{z}{(z-a)^2}$ d) $\frac{z}{(z+a)^2}$

10. The Z – Transform of the function $(-2)^n$ is……………..

PART B (10 x 2 = 20 Marks)

(Not more than 40 words)

11. Form the partial differential equation by eliminating a and b from $2z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$.

12. Find the complete solution of $p^2+q^2=1$.

13. State the assumptions made in the derivation of one dimensional wave equation.

14. A rod 30 cm long has its ends A and B kept at $20^0 C$ *and* $80^0 C$ respectively until steady state conditions prevail. Find the steady state temperature in the rod.

15. Does f(x) = tan x possess a Fourier expansion? Justify.

16. What do you mean by Harmonic Analysis?

17. If $F(s)=F[f(x)]$ Prove that $F[f(x-a)]=e^{ias}F(s)$

18. State the convolution theorem for Fourier transforms.

19. Find the transfer function of the difference equation $x_{n+2}-5x_{n+1}+6x_n=0$, given that $x_0=x_1=0$ using Z-transform.

20. Determine the inverse Z-transform of $\frac{1}{(z-1)(z-2)}$.

PART C (5 x 14 = 70 Marks)

(Not more than 400 words)

Q.No. 21 is Compulsory

21. An infinitely long uniform plate is bounded by two parallel edges and an end at right angle to them. The breadth of the edge x = 0 is π, this end is maintained at temperature as $u=K(\pi y-y^2)$ at all points while the other edges are at zero temperature. Find the temperature u(x, y) at any point of the plate in steady state.

22. a) (i) If $z = f(x+ay) + \varphi(x-ay)$ then prove that $\frac{\partial^2 z}{\partial y^2} = a^2\left(\frac{\partial^2 z}{\partial x^2}\right)$. (7)

(ii) Solve the equation $(pq - p - q)(z - px - py) = pq$. (7)

(OR)

b) (i) Determine the solution of the form $\varphi(u,v) = 0$ if $p + q = x + y + z$ (7)

(ii) Solve $\left(D_x^3 - 7D_xD_y^2 - 6D_y^3\right)z = \sin(x+2y) + e^{3x+y}$. (7)

23. a) (i) Obtain the Fourier series expansion of $f(x) = \begin{cases} 1, 0<x<1 \\ x, 1<x<2 \end{cases}$. Hence find the value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ...$ (7)

(ii) Determine the half range cosine series of $f(x) = x\sin x$ in $(0,\pi)$. (7)

(OR)

b) (i) Find the Half – range sine series of $f(x) = x \, in \; 0<x<l$ and deduce the series $\sum_{1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$. (7)

(ii) Expand f (x) in a Fourier series upto 2nd harmonic using the following table. (7)

x	0	1	2	3	4	5
$f(x)$	9	18	24	28	26	30

24. a) (i) Find the Fourier transform of $e^{-a^2x^2},\ a>0.$ Hence show that the function $e^{-\frac{x^2}{2}}$ is self – reciprocal. (10)

(ii) State and prove Modulation Theorem. (4)

(OR)

b) (i) Determine the Fourier transform of $f(x) = \begin{cases} 1, |x| < a \\ 0. |x| > a > 0 \end{cases}$. Hence evaluate (i) $\int_0^{\infty}\frac{\sin x}{x}dx$ (ii) $\int_{-\infty}^{\infty}\frac{\sin as \cos sx}{s}ds$. (7)

(ii) Obtain the Fourier Cosine transform of e^{-ax} & e^{-bx} and hence show that $\int_0^{\infty}\frac{1}{\left(x^2+a^2\right)\left(x^2+b^2\right)}dx = \frac{\pi}{2ab(a+b)}$ by using Parseval's identity. (7)

25. a) (i) Find the Z – Transform of 1) $\frac{1}{n(n+1)}$ 2) $a^n \sin n\theta$. (7)

(ii) Find the inverse Z – Transform of $\frac{8z^2}{(2z-1)(4z-1)}$ by using convolution theorem. (7)

(OR)

b) (i) Solve the equation $y(x+2)+4y(x+1)+4y(x)=x$, given that $y(0)=0\, and\ y(1)=1$.

****