

MCA DEGREE EXAMINATIONS: NOV/DEC 2014

(Regulation 2009)

Second Semester

MASTER OF COMPUTER APPLIATIONS

MAT509: Mathematical Foundations Of Computer Science

Time: Three Hours

Maximum Marks: 100

Answer all the Questions:-

PART A (10 x 2 = 20 Marks)

1. Define Rank of a Matrix.
2. Find the sum and product of the eigen values of $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.
3. Let $A = \{1,2,3,4\}$, $B = \{1,4,9,16\}$ and the relation $R = \{(1,1), (2,4), (3,9), (4,16)\}$. Draw the relation graph.
4. Define Bijective function.
5. Let P be “Ravi speaks Tamil” and q be “Ravi speaks Hindi”. Give a simple verbal sentence for $\sim P \vee \sim q$.
6. Write the symbolic form of “Every student in this class has studied Logic”.
7. Define Regular language.
8. Let G be the grammar with vocabulary $V = \{S, A, a, b\}$, set of terminals $T = \{a, b\}$, starting symbol S, and production $P = \{S \rightarrow aA, S \rightarrow b, A \rightarrow aa\}$. What is the language of this grammar?.
9. Draw the state diagram for the NFA for which the state table is given below, for which the accepting states are S_1 and S_2 .

State	f	
	a	b
S_0	S_0, S_1	S_2
S_1	\emptyset	S_1
S_2	S_1, S_2	\emptyset

10. Write the difference of DFA and NFA.

PART B (5 x 16 = 80 Marks)

11. a) (i) Find the Rank of the matrix $A = \begin{pmatrix} 4 & -5 & 1 & 2 \\ 3 & 1 & -2 & 9 \\ 1 & 4 & 1 & 5 \end{pmatrix}$. (6)

(ii) Verify Cayley –Hamilton theorem for $\begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$. Hence find its inverse (10)

(OR)

b) (i) Test the consistency of the following equations and if possible find the solution of $4x - 2y + 6z = 8; x + y - 3z = -1; 15x - 3y + 9z = 21$. (6)

(ii) Find the eigen values and eigen vectors of $\begin{pmatrix} 4 & -20 & -10 \\ -2 & 10 & 4 \\ 6 & -30 & -13 \end{pmatrix}$. (10)

12. a) (i) Prove that $(A \cap B) \cup C = A \cap (B \cup C)$ iff $C \subseteq A$. (8)

(ii) Among 50 students in a class, 26 got an A in the first examination and 21 got A in the second examination. If 17 students did not get an A in either examination, how many students got an A in both examinations? (8)

(OR)

b) (i) Let R denote a relation on the set of ordered pairs of positive integers such that $(x, y)R(u, v)$ iff $xv = yu$. Show that R is an equivalence relation. (8)

(ii) State whether the function $f(x) = 5x^2 + 7$ is injection, surjection or bijection on R, the set of real numbers. (8)

13. a) (i) Show that $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a tautology. (8)

(ii) Show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$. (8)

(OR)

b) (i) Obtain PDNF of $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$. Also find PCNF. (8)

(ii) Show the following argument is valid. (8)

“My father praises me only if I can be proud of myself. Either I do well in sports or I cannot be proud of myself. If study hard, then I cannot do well in sports. Therefore, if father praises me, then I do not study well”.

14. a) (i) Find a phrase structure grammar to generate $\{0^n 1^n 2^n / n = 0, 1, 2, \dots\}$ (8)
(ii) Explain the types of phrase-structure grammars. (8)

(OR)

- b) (i) Show that the following grammars are ambiguous. (10)

1. $S \rightarrow SS, S \rightarrow a, S \rightarrow b$

2. $S \rightarrow aAb, S \rightarrow abSb, S \rightarrow a, A \rightarrow bS, A \rightarrow aAAb$

- (ii) Give a phrase-structure grammar that generates the set of all strings that begin with an even number of 1s and end with a 0. (6)

15. a) (i) Draw the state diagram for the NFA for which the stable is given below and the accepting states are S_1 and S_3 . Find the language accepted by this NFA. (6)

State	f	
	a	b
S_0	S_2	S_1
S_1	S_1, S_2	S_3
S_2	\emptyset	\emptyset
S_3	S_2, S_3	S_2

- (ii) Find the DFA equivalent to the NFA for which the state table is given below and S_2 is the accepting state. (10)

State	f	
	a	b
S_0	S_0, S_1	S_2
S_1	S_0	S_1
S_2	S_1	S_0, S_1

(OR)

- b) (i) Construct an FSA that accepts all strings over $\{a, b\}$ that contain m 'a's where m is a multiple of 3. (6)
(ii) Construct a deterministic finite automation (FA) equivalent to an NFA with the Transition diagram given in the figure below (10)


