Register Number: ………………..

B.E DEGREE EXAMINATIONS: NOV 2015

(Regulation 2009)

Seventh Semester

COMPUTER SCIENCE ENGINEERING

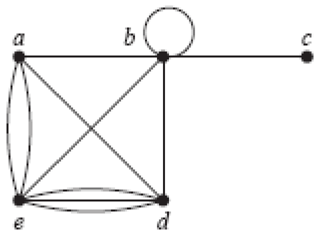
MAT111 Graph Theory

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1.



The degree of the vertex ‘e’ of the graph is ______

a) 4 b) 6

c) 0 d) 5

2. The length of a Hamiltonian path(if it exists) in a connected graph of n vertices is _______

a) $n+1$ b) n^2

c) $2n$ d) $n-1$

3. The value of the postfix expression 7 2 3 ∗ − 4 ↑ 9 3/+ is______

a) 4 b) 3

c) 7 d) 9

4. A tree with 6 vertices has ________ edges.

a) 6 b) 4

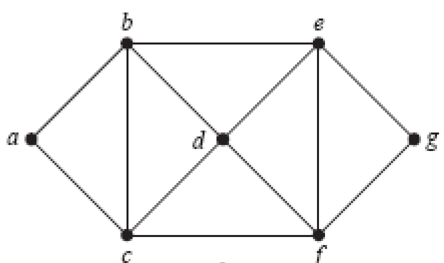
c) 5 d) 7

5. A graph consisting of only isolated vertices is ________

a) 2-chromatic b) 3-chromatic

c) 1-chromatic d) 4-chromatic

6. The chromatic numbers of the graph G is _____

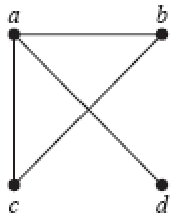


G

a) 2 b) 3

c) 4 d) 5

7. The adjacency matrix representing the given graph is______



a) $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}$ b) $\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

c) $\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$ d) $\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}$

8. The definition of _______matrix makes no provision for parallel edges.
   a) Path   b) circuit
   c) incidence   d) adjacency

9. The number edges must be removed from a connected graph with *n* vertices and *m* edges to produce a spanning tree is ___
   a) $m - n - 1$ b) $m + n - 1$
   c) $m + n + 1$ d) $m - n + 1$

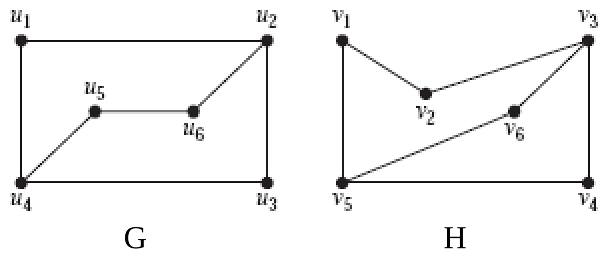
10. The method used for finding shortest spanning tree is _______
   a) Kruskal algorithm   b) Simplex algorithm
   c) Hungarian algorithm   d) Min-cut algorithm

PART B (10 x 2 = 20 Marks)

11. Define connected graph.
12. Give an example of non Eulerian graph.
13. Write any two properties of tree.
14. Determine the value of the prefix expression $+-*235/\uparrow 234$.
15. Prove that K_5 is not a planar graph.
16. Define planar graph.
17. Can the 4 x 4 identity matrix be the adjacency matrix of a graph? Justify.
18. Define path matrix of a graph.
19. Draw a graph with eight spanning trees.
20. State conservation of flow in a network.

PART C (5 x 14 = 70 Marks)

21. a) (i) Determine whether the graphs G and H displayed given below are isomorphic. (6)



(ii) Show that a graph with at least two vertices is Eulerian if and only if it is connected and every vertex is even. (8)

(OR)

b) (i) Prove that a simple graph with n vertices and k components have at most $\frac{(n-k)(n-k+1)}{2}$ edges. (8)

(ii) Show that the number of vertices of odd degree in a graph is always even. (6)

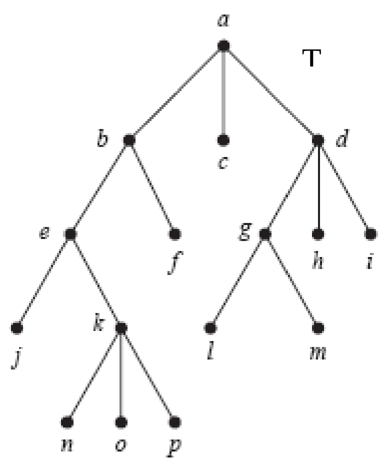
22. a) (i) Let G be a graph. Then prove that the following statements are equivalent. (7)

1. G is a tree.
2. G is connected and contains no cycles.
3. Between any two vertices of G there is precisely one path.

(ii) Show that a tree which contains more than one vertex must contain at least two vertices of degree 1. (7)

(OR)

b) (i) In which order does an inorder traversal visit the vertices of the ordered rooted tree T in the following. (7)



(ii) Show that every tree has either one or two centers. (7)

23. a) (i) Let G be a connected plane graph with V vertices, E edges and R regions. Then prove that V - E + R = 2. (7)

(ii) Let $\Delta(G)$ be the maximum of the degree of the vertices of a graph G. Then show that the chromatic number of a graph G is $\leq 1+\Delta(G)$. (7)

(OR)

b) (i) Let G be a planar graph with $V \geq 3$ vertices and E edges. Then prove that $E \leq 3V-6$. (7)

(ii) Prove that every tree with two or more vertices is 2- chromatic. (7)

24. a) (i) If A(G) is an incidence matrix of a connected graph G with n vertices then prove that the rank of A(G) is $n-1$ (7)

(ii) Let A and B be the incidence matrix and circuit matrix respectively, whose columns are arranged using the same order of edges. Then show that every row of B is orthogonal to every row of A. (7)

(OR)

b) (i) If B is a circuit matrix of a connected graph G with e edges and n vertices then Show that rank of B is $e-n+1$. (7)

(ii) Explain the following with example: (7)

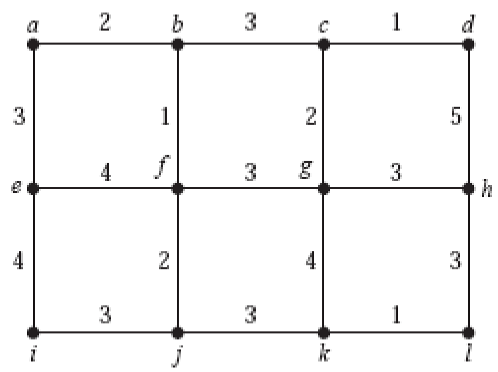
1. Adjacency matrix
2. Path matrix

25. a) (i) Prove that a simple graph is connected if and only if it has a spanning tree. (8)

(ii) Determine the number of spanning trees of a connected graph G whose adjacency matrix is $A=\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}$. (6)

(OR)

b) (i) Use Prim's algorithm to find a minimum spanning tree in the graph. (8)



(ii) Prove that the maximum flow possible between two vertices a and b in a network is equal to the minimum of the capacities of all cut sets with respect to a and b. (6)

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