KUMARAGURU
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Register Number: ………………..

B.E DEGREE EXAMINATIONS: NOV 2015

(Regulation 2009)

Seventh Semester

COMPUTER SCIENCE AND ENGINEERING

MAT112: Partial Differential Equations And Their Solution

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. The PDE of the family of spheres having their centres on the line x = y = z is

   a) $(1+q)(x+zp)=(1+p)(y+zq)$ b) $x+zp=y+zq$

   c) $q(x+zp)=p(y+zq)$ d) $p(x-zp)=q(y-zq)$

2. The singular integral $3z=xy-x^2-y^2$ of a surface whose tangent planes are given by the complete integral

   a) $z=px+qy+p^2+pq+q^2$ b) $z=ax+by+a^2+ab+b^2$

   c) $z=px+qy+p^2+q^2$ d) $z=p^2+q^2+pq$

3. If $\bar{A}\ u_{\xi\xi}+\bar{B}\ u_{\xi\eta}+\bar{C}\ u_{\eta\eta}+\bar{D}\ u_{\xi}+\bar{E}\ u_{\eta}+\bar{F}u=\bar{G}$ be the canonical form of $Au_{xx}+Bu_{xy}+Cu_{yy}+Du_x+Eu_y+Fu=G$ then $\bar{B}$ is

   a) $2A\xi_x\eta_y+B(\xi_x\eta_y+\xi_y\eta_x)+2C\xi_x y$ b) $A\,{\xi_x}^2+B\xi_x\xi_y+C{\xi_y}^2$

   c) $A\,{\eta_x}^2+B\eta_x\eta_y+C{\eta_y}^2$ d) $A\xi_{xy}+B\xi_{xx}+D\xi_x+E\xi_y$

4. The PDE $y^3u_{xx}-(x^2-1)u_{yy}=0$ is

   a) parabolic in $\{(x,y): x<0\}$ b) hyperbolic in $\{(x,y): y>0\}$

   c) elliptic in $\mathbb{R}^2$ d) parabolic in $\{(x,y): x>0\}$

5. In the steady state, the temperature satisfies

   a) Laplace Equation b) Poisson Equation

   c) Euler's Equation d) Cauchy's Equation

6. The polar form of the Laplace equation $\nabla^2 u=0$ is

   a) $u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta}=0$ b) $u_{\theta\theta}+\frac{1}{\theta}u_\theta+\frac{1}{\theta^2}u_{rr}=0$

c) $u_{\theta\theta} + \frac{1}{r}u_r + \frac{1}{r^2}u_{rr} = 0$ d) $u_{rr} + \frac{1}{r}u_r + r^2 u_{\theta\theta} = 0$

7. The Laplace transform of $\frac{1}{\sqrt{t}}$ is

a) $\frac{\pi}{s}$ b) $\sqrt{\pi / s}$

c) $\sqrt{\pi}$ d) $\sqrt{\pi} / s$

8. In the Millin-Fourier Integral $f(t)$ is defined as

a) $\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{-st}F(s)ds$ b) $\frac{1}{2\pi i}\int_{\gamma-\infty}^{\gamma+\infty} e^{st}F(s)ds$

c) $\frac{1}{2\pi i}\int_{\gamma-\infty}^{\gamma+\infty} e^{-st}F(s)ds$ d) $\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{st}F(s)ds$

9. If $u(x,t)$ and $\frac{\partial u}{\partial x}$ vanish as $x \to \pm\infty$ then $\mathcal{F}\left[\frac{\partial^2 u}{\partial x^2}; x \to \alpha\right]$

a) $-i\,\alpha^2 U_{xx}(t,\alpha)$ b) $(-1)^2 (i\,\alpha)^2 U_{xx}(t,\alpha)$

c) $-i\,\alpha^2 U_{xx}(\alpha,t)$ d) $(-1)^2 (i\,\alpha)^2 U_{xx}(\alpha,t)$

10. The error function erf (y) is defined as

a) $\frac{2}{\sqrt{\pi}}\int_0^y e^{-u^2}du$ b) $\frac{2}{\pi}\int_0^y e^{-u^2}du$

c) $\frac{\sqrt{\pi}}{2}\int_0^u e^{-y^2}du$ d) $\frac{\sqrt{\pi}}{2}\int_0^y e^{-\sqrt{u}}du$

PART B (10 x 2 = 20 Marks)

11. Form the partial differential equation by eliminating 'a' and 'b' from $z^2 \cot^2 \alpha = (x-a)^2 + (y-b)^2$

12. Solve $\frac{\partial^2 u}{\partial x \partial t} = e^{-t}\cos x$.

13. Classify the Partial differential equation $y^2 u_{xx} - 2xyu_{xy} + x^2 u_{yy} + 2u_x - 3u = 0$.

14. Find the characteristic of the equation $u_{xx} + 2\,u_{xy} + Sin^2(x)u_{yy} + u_y = 0$.

15. Define the Neumann Problem for a circle.

16. Write the possible solutions of $\nabla^2 u = 0$.

17. Find the inverse Laplace transform of $\frac{e^{-4s}}{(s+5)(s-5)}$.

18. State the Dirac delta function

19. Prove that $\mathcal{F}\left[\frac{\partial u(x,t)}{\partial x}; x \to \alpha\right] = -i\,\mathcal{F}[u(x,t); x \to \alpha]$ if $u(x,t)$ vanish as $x \to \pm\infty$.

20. State Faltung theorem.

PART C (5 x 14 = 70 Marks)

21. a) i) Solve $z = px + qy + \sqrt{1 + p^2 + q^2}$ (7)

ii) Find the complete integral of $p^2 + q^2 = x + y$ (7)

(OR)

b) i) Solve $(D^2 - 6DD' + 9D'^2)z = x^2y^2 + \cos(3x + y)$ (7)

ii) Solve $x\left(z^2 - y^2\right)p + y\left(x^2 - z^2\right)q = z\left(y^2 - x^2\right)$ (7)

22. a) Reduce the Tricomi equation $u_{xx} + x\,u_{yy} = 0, x \neq 0\ for\ all\ x, y$ to canonical form.

(OR)

b) Reduce the equation $3u_{xx} + 10u_{xy} + 3u_{yy} = 0$ to a canonical form and hence solve it.

23. a) A thin rectangular homogeneous thermally conducting plate lies in the xy-plane defined by $0 \leq x \leq a, 0 \leq y \leq b$. The edge $y = 0$ is held at the temperature $Tx(x - a)$, where T is constant, while the remaining edges are held at 0°. The other faces are insulated and no internal sources and sinks are present. Find the steady state temperature inside the plate.

(OR)

b) Solve interior Dirichlet Problem for a circle.

24. a) Using Laplace transform , Solve the following IBVP:

PDE : $u_t = u_{xx}\,, 0 < x < l\,, t > 0$

BCs : $u(x,0) = 1\ \,, u(l,t) = 1\,, t > 0$

IC : $u(x,0) = 1 + sin\pi x\,,\ 0 < x < l$

(OR)

b) Using the Laplace Transform method, solve the IBVP $\frac{\partial^2 u}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}-\cos wt$, $0\le x\le\infty,\ t\ge 0$ subject to the initial and boundary conditions, u is bounded as $x\to\infty$, $u(0,\ t)=0$, $\frac{\partial u}{\partial t}(x,0)=u(x,0)=0$.

25. a) Determine the temperature distribution in the semi-infinite medium x ≥ 0, when the end x = 0 is maintained at zero temperature and the initial temperature distribution is f(x).

(OR)

b) Solve the boundary value problem in the half plane > 0 , described by

PDE : $u_{xx}+u_{yy}=0\ ,-\infty<x<\infty\ ,y>0$

BCs : $u(x,0)=f(x)\ ,\ -\infty<x<\infty$

u is bounded as $y\to\infty$; u and $\frac{\partial u}{\partial x}$ both vanish as $|x|\to\infty$.

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