KUMARAGURU college of technology
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M.E DEGREE EXAMINATIONS: JAN 2015

(Regulation 2014)

First Semester

APPLIED ELECTRONICS

P14MAT108**:** Applied Algebra

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. The system of equations $x+2y=10, 2x+4y=20$ has [K_3]

   a) Unique solution b) No solution

   c) Infinite solution d) Trivial solution

2. Consider the following statements [K_2]

   1) Each matrix is row equivalent to one and only one reduced echelon matrix.
   2) The equation Ax=b has a solution if and only if b is a linear combination of the columns of A.
   3) The homogeneous equation Ax=0 has nontrivial solution if and only if the equation has at least two free variable.
   4) The columns of a matrix A are linearly independent if and only if the equation Ax=0 has only the non trivial solution.

   Which of these statements are not correct?

   a) 1,2 b) 3,4

   c) 1,2,4 d) 1,3,4

3. Assertion (A) : If A is an invertible matrix then $A^{-1}$is invertible. [K_2]

   Reason (R): Every square matrix is invertible.

   a) both A and R are individually true but R is the correct explanation of A b) both A and R are individually true but R is not the correct explanation of A

   c) A is true but R is false d) A is false but R is true

4. Consider the following statements [K_3]

   1) If A is a triangular matrix then det(A) is the product of the entries on the main diagonal of A.
   2) det(A+B)= det(A)+det(B).
   3) Rank of the matrix A is the order the matrix A.
   4) Cramer's rule is used to solve the difference equation.

   Which of these statements are not correct?

   a) 2,3 b) 1,3,4

   c) 1,2,3,4 d) 2,3,4

5. Match the following: [K_1]

List I	List II
A) T(au+bv)=aT(u)+bT(v)	1) Orthogonal transformation
B) $Au = 0$	2) Linearity
C) T(cu)=cT(u)	3) null space
D) $N^T AN=D$	4) scalar multiplication

Codes:

	A	B	C	D
a.	2	1	4	3
b.	3	4	1	2
c.	2	3	4	1
d.	4	3	2	1

6. Match the following: [K_1]

List I	List II
A) Product of eigen values is equal to	1)Trace of A
B) If atleat one eigne value is zero then	2) $\|A\| \neq 0$
C)If the latent roots are positive then	3) $\|A\| = 0$
D) Sum of the eigen values is equal to	4) $\|A\|$

Codes:

	A	B	C	D
a.	4	3	2	1
b.	3	2	4	1
c.	2	1	4	3
d.	4	3	1	2

7. Assertion (A) : If W is a subspace spanned by an orthonormal set then it forms an orthonormal basis for W. [K_4]
Reason (R): The orthogonal set of unit vectors is linearly independent.

a) both A and R are individually true but R is the correct explanation of A
b) both A and R are individually true but R is not the correct explanation of A
c) A is true but R is false
d) A is false but R is true

8. The steps involved in reducing the given matrix to diagonal form through orthogonal transformation. [K_4]
1. Check the given matrix is symmetric.
2. Find P such that $P^T AP = D$.
3. Check the vectors are linearly independent and pairwise orthogonal.
4. Find the latent roots and latent vectors.

a) 1-2-3-4
b) 2-3-1-4
c) 1-4-3-2
d) 4-1-2-3

9. Memory less property holds good for [K_1]

a) Binomial distribution
b) Poisson distribution
c) Geometric distribution
d) Normal distribution

10. The steps involved in computing correlation co efficient of two variables. [K_2]
1. Identify suitable formulae.

2. Check the variables are discrete or continuous.
3. Calculate necessary values.
4. Interpret the result.

a) 1-2-4-3 b) 2-1-3-4

c) 3-4-1-2 d) 2-1-4-3

PART B (10 x 2 = 20 Marks)

11. Let $u = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$. Display the vectors $u, 2u, -\frac{2}{3}u$ on a graph. [K_2]

12. List all the iterative methods to solve the system of linear equations. [K_1]

13. If A and B are n x n matrices and the equation (AB)x = 0 has a nontrivial solution then assess the nature of matrix AB. [K_1]

14. Determine the value of "a" such that ρ(A) = 2 where A = $\begin{pmatrix} 2 & 1 & 1 & 2 \\ 3 & 0 & -1 & 4 \\ 7 & 2 & a & 8 \end{pmatrix}$. [$K_2$]

15. Say True or False: If S spans V(nonzero finite dimensional vector space) and if T is a subset of V that contains more vectors than S, then T is linearly dependent. Justify. [K_1]

16. Define characteristic equation. [K_1]

17. Compute the distance between the vectors u=(7, 1) and v=(3, 2). [K_3]

18. Show that $\{u_1, u_2, u_3\}$ is an orthogonal set, where $u_1 = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}, u_2 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, u_3 = \begin{pmatrix} -1/2 \\ -2 \\ 7/2 \end{pmatrix}$. [$K_4$]

19. Find E(X) for a Poisson random variable X, if $E(X^2)=6$. [K_3]

20. State the basic properties of Standard Normal Curve. [K_2]

PART C (6 x 5 = 30 Marks)

21. Solve the following set of equations, using row-echelon form. [K_4]

$x + 2y - z = 6$
$3x + 8y + 9z = 10$
$2x - y + 2z = -2$

22. Let $A = \begin{pmatrix} -3 & 1 & 2 \\ 1 & -4 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}$ verify that [K_3]

$AB = col_1(A)row_1(B) + col_2(A)row_2(B) + col_3(A)row_3(B)$.

23. Use coordinate vectors to verify that the polynomials $1 + 2t^2, 4 + t + 5t^2, 3 + 2t$ are linearly dependent in P_2. [K_2]

24. [K_5]

Let $a=\begin{pmatrix}-2\\1\end{pmatrix}, b=\begin{pmatrix}-3\\1\end{pmatrix}, c=\begin{pmatrix}4/3\\-1\\2/3\end{pmatrix}$, *and* $d=\begin{pmatrix}5\\6\\-1\end{pmatrix}$

I. Compute $\frac{a\bullet b}{a\bullet a}$ *and* $\left(\frac{a\bullet b}{a\bullet a}\right)a$

II. Find a unit vector u in the direction of c.

III. Justify: d is orthogonal to c

IV. Is d orthogonal to the unit vector u? If so explain.

25. Find the MGF of exponential distribution and hence find mean and variance. [K_4]

26. Examine whether the given vectors are linearly independent. If they are linearly dependent, find the relationship between them. [K_5]

$X_1=(1,-1,-2,-4), X_2=(2,3,-1,-1), X_3=(3,1,3,-2)$ *and* $X_4=(6,3,0,-7)$

PART D (4 x 10 = 40 Marks)

27. Solve the equation $Ax=b$ by using the LU factorization, [K_5]

given $A=\begin{pmatrix}3&-7&-2\\-3&5&1\\6&-4&0\end{pmatrix}$, $b=\begin{pmatrix}-7\\5\\2\end{pmatrix}$.

28. Suppose a particle is moving in a planar force field and its position vectors x satisfies $x'=Ax$ *and* $x(0)=x_0$ where $A=\begin{pmatrix}4&-5\\-2&1\end{pmatrix}$, $x_0=\begin{pmatrix}2.9\\2.6\end{pmatrix}$. Solve this initial value problem. [K_5]

29. [K_6]

Construct a singular value decomposition for the matrix $A=\begin{pmatrix}2&2&-2\\2&2&-2\\-2&-2&6\end{pmatrix}$.

30. The joint cdf of a bivariate random variable (X, Y) is given by [K_6]

$$f_{xy}(x,y)=\begin{cases}(1-e^{-\alpha x})(1-e^{-\beta y}), & x\geq 0, y\geq 0, \alpha,\beta>0\\ 0, & otherwise\end{cases}$$

(a) Find the marginal cdf's of X and Y.
(b) Show that X and Y are independent.
(c) Find P(X ≤ 1, Y ≤ 1), P(X ≤ 1), P(Y > 1) and P(X > x, Y > y).

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