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Register Number:………….……..….

M.E DEGREE EXAMINATIONS: JAN 2015

(Regulation 2014)

First Semester

COMPUTER SCIENCE AND ENGINEERING

P14MAT109: Advanced Computational Mathematics

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. Consider the following statements [K2]

1) The canonical form of the wave equation $u_{tt} = c^2 u_{xx}$ when $\xi = x - ct$, $\eta = x + ct$ is $u_{\eta\eta} = 0$

2) The type of partial differential equation $\frac{\partial^2 f}{\partial x^2} + 2\frac{\partial^2 f}{\partial x \partial y} + 4\frac{\partial^2 f}{\partial y^2} = 0$ is Parabolic.

3) A proper solution of steady state heat flow in two dimensions, if the non-zero boundary values is prescribed on x=a is $u(x,y) = (Ax+B)(Cy+D)$.

4) The partial differential equation that represents steady state heat flow in polar coordinates is $u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$.

Which of these statements are correct?

a) 2,3,4 b) 1,3

c) 2,4 d) 1,2

2. Match list I with list II (answer using the following codes) [K1]

List I	List II
A) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$	1) 1D wave equation
B) $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$	2) 1D heat equation
C) $\frac{\partial^2 u}{\partial x^2} = 2\frac{\partial u}{\partial t}$	3) Hyperbolic equation
D) $\frac{\partial^2 u}{\partial x^2} = \frac{1}{C^2} \frac{\partial^2 u}{\partial t^2}$	4) Laplace equation

Codes:

	A	B	C	D
a.	4	3	2	1
b.	3	2	4	1
c.	2	3	1	4
d.	4	1	2	3

3. The steps involved in finding the solution of boundary value problem by Ritz Method. [K_3]
   1. Determine the corresponding functional of the boundary value problem.
   2. Check the boundary conditions.
   3. Identify the value of constant in the trial solution.
   4. Choose the trial solution.

   Choose the correct sequence

   a) 1-2-3-4 b) 1-2-4-3

   c) 1-3-2-4 d) 1-3-4-2

4. Assertion (A): A differential equation with boundary conditions is called a boundary value problem. [K_2]

   Reason (R): Any equations with boundary conditions are called boundary value problems.

   a) both A and R are individually true but R is the correct explanation of A
   b) both A and R are individually true but R is not the correct explanation of A
   c) A is true but R is false
   d) A is false but R is true

5. Match list I with list II (answer using the following codes)

List I	List II
A) sum of the eigen values of a matrix	1) determinant of the matrix is zero
B) one of the eigen values is zero	2) determinant of the matrix
C) product of the eigen values of a matrix	3) real symmetric matrix
D) eigen values are real	4) trace of the matrix

Codes:

	A	B	C	D
a.	4	3	2	1
b.	3	2	4	1
c.	2	3	1	4
d.	4	1	2	3

6. The number of rotations required for the Given’s method are equivalent to [K_2]

   a) The number of diagonal elements
   b) The number of non diagonal elements
   c) The number of non – tri-diagonal elements
   d) The number of tri-diagonal elements

7. [K_2]
   1. Leibmann’s iteration method is used to solve Poisson equation.
   2. Leibmann’s iteration method is used to solve one dimensional wave equation.
   3. Crank Nicholson method is an explicit method.
   4. Bender Schmidt method is an explicit method.

   Which of the following statements are not correct?

   a) 1,3 b) 1,4

c) 2,4 d) 2,3

8. Assertion (A) : Laplace equation is solved by Leibmann's method [K_2]

Reason(R) : partial differential equations can be solved by the method of finite differences.

a) both A and R are individually true but R is the correct explanation of A
b) both A and R are individually true but R is not the correct explanation of A
c) A is true but R is false
d) A is false but R is true

9. All shape functions of an n-noded element are of polynomial of degree______ [K_2]

a) n
b) n-1
c) 0
d) n^2

10. The steps involved in finding the solution of boundary value problem by Galerkin's FEM Method. K_3

1. Obtain finite element equations.
2. Apply the boundary conditions.
3. Convert the given equation into weak form.
4. Find the solution of finite element equation.

a) 1-2-3-4
b) 3-2-1-4
c) 3-1-2-4
d) 1-3-2-4

PART B (10 x 2 = 20 Marks)

11. Examine the nature of the partial differential equation $r + s - 2t - p - 2q = 0$. [K_3]

12. Write down Monge's subsidary equations. [K_2]

13. Determine the extermals of functional $I(y) = \int_1^3 (3x - y)y\, dx$ that satisfy the boundary conditions y (1) =1, y (3) =9/2. [K_3]

14. Define geodesic of a surface. [K_2]

15. Find the numerically largest eigen value of the matrix $A = \begin{pmatrix} 4 & 1 \\ 1 & 3 \end{pmatrix}$ [K_2]

16. List out the uses of Sylvester's expansion theorem. [K_2]

17. What is the Schmidt finite difference scheme to solve $u_t = c^2 u_{xx}$. [K_4]

18. State explicit finite difference scheme for one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$. [$K_2$]

19. Explain finite element method. [K_4]

20. What are different types of elements in FEM. [K_2]

PART C (6 x 5 = 30 Marks)

21. Reduce the equation $u_{xx} + \frac{2N}{x} u_x = \frac{1}{a^2} u_{tt}$, where N and a are constants, to a canonical form [K_5]

22. Identify the plane curve of fixed perimeter and maximum area. [K_4]

23. Apply Given's method, to reduce the matrix $A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 4 & 2 \\ 3 & 2 & 3 \end{pmatrix}$ to the tri-diagonal form. [K_4]

24. Determine the solution of the equation $u_{xx} = u_t$, subject to the conditions $u(x,0) = \sin \pi x,\ 0 \le x \le 1;\ u(0,t) = u(1,t) = 0,$ by applying Bender-Schmidt method. Carryout computations for two levels, taking h=1/3, k=1/36. [K_5]

25. Certain problem of one dimensional heat transfer is generated by the equation [K_5]

$\frac{d^2y}{dx^2} + y + 1 = 0$ and the boundary conditions $y = 1\ at\ x = 0\ and\ \frac{dy}{dx} = 1\ at\ x = 1$. Solve this problem by Galerkin's method.

26. Develop the characteristic polynomial and inverse of the matrix $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & -1 & 2 \end{pmatrix}$ using Faddeev- Leverrier method. [K_4]

PART D (4 x 10 = 40 Marks)

27. V is a function of $r\ and\ \theta$ satisfying the equation $\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} = 0$ when the region of the plane bounded by $r = a, r = b, \theta = 0, \theta = \frac{\pi}{2}$. Its value along the boundary $r = a\ is\ \theta\left(\frac{\pi}{2} - \theta\right)$, along the other boundaries is zero. Find $V(r,\theta)$. [K_4]

28. Employ Rayleigh Ritz method, to solve the boundary value problem $y'' - 6x = 0,\ y(0) = 0,\ y(1) = 1$. [K_3]

29. Evaluate the pivotal values of the equation $u_{tt} = 16u_{xx}$ taking $\Delta x = 1$ upto $t = 1.25$. The boundary conditions are $u(0,t) = u(5,t) = 0, u_t(x,0) = 0$ and $u(x,0) = x^2(5-x)$. [K_5]

30. Using Galerkin's FEM, obtain the solution of the boundary value problem $\frac{d^2u}{dx^2} - x = 0, 0 \le x \le 2$ with the boundary conditions: u=0 at x=0 and x=2. [K_5]

Discretize the domain into two equal elements.

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