Register Number:……………..

KUMARAGURU
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B.E/B.TECH DEGREE EXAMINATIONS: JUNE 2016

(Regulation 2015)

Second Semester

U15MAT201: ENGINEERING MATHEMATICS - II

(Common to All Branches)

COURSE OUTCOMES

CO1: To understand double and triple integrations and enable them to find area and volume using multiple integrals.

CO2: To know the basics of vector calculus comprising gradient, divergence and curl and line, surface and volume integrals.

CO3: To understand analytic functions of complex variables and conformal mappings.

CO4: To know the basics of residues, complex integration and contour integration.

CO5: To understand Laplace transform and use it to represent system dynamic models and evaluate their time responses.

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions

PART A (10 x 1 = 10 Marks)

1. The Value of $\int_0^{\pi/2}\int_0^{\sin\theta} r dr\, d\theta$ CO1 [K_1]

   a) π/4 b) π/6

   c) π/8 d) 2π/5

2. Consider the following statements CO3 [K_2]

   (1) If $f(z)=u+iv$ is analytic, the first order partial derivatives of u and v are continuous

   (2) If the real part of an analytic function $f(z)$ is constant, then $f(z)$ is a constant

   (3) If $f(z)=u+iv$ is analytic, then only v satisfies Laplace equation

   (4) If $f(z)=u+iv$ is analytic, then u and v are harmonic conjugate of each other

   Which of these statements are correct?

   a) 1,3 b) 1,2

   c) 2,4 d) 3,4

3. By Green's theorem $\int_c \left(x^2dx + xydy\right)$, where c is the curve in XY plane given by $x=0, y=0, x=a, y=a\left(a>0\right)$ is CO2 [K_3]

a) a^2

b) $\frac{a^3}{2}$

c) $\frac{a^3}{3}$

d) a^3

4. The image of the circle $|z|=9$ under the transformation $w=5z$ is CO3 [K_2]

a) $u^2+v^2=225$

b) $u^2+v^2=15$

c) $u^2+v^2=45$

d) $u^2+v^2=81$

5. Consider the singularities of $\frac{e^z}{z^2+4}$ CO4 [K_2]

(1) $z=0$ is an essential singularity

(2) $z=\pm 2$ are simple a poles

(3) $z=2i$ is a simple pole

(4) $z=-2i$ is a simple pole

Which of these statements about nature of singularities are correct?

a) 1,3

b) 3,4

c) 1,2

d) 2,3

6. The steps involved in evaluating the contour integrals are CO4 [K_2]

(1) Apply Cauchy's Residue theorem

(2) Identify the function $f(z)$

(3) Determine the residues at the poles

(4) Find the poles of $f(z)$ and test which are inside c

a) 2-3-4-1

b) 1-2-4-3

c) 1-2-3-4

d) 2-4-3-1

7. If $f(t)=t^2e^{-3t}$, then $\lim_{t\to\infty} f(t)=$ CO5 [K_3]

a) 0

b) ∞

c) 2/9

d) 2/3

8. Examine the two statements carefully and select the answer using the codes given below: CO1 [K_3]

Assertion (A): Double integral is used to find area between the curves

Reason (R):The variables x and y in the formula for area may be in any order

a) Both A and R are individually true and R is the correct explanation of A

b) Both A and R are individually true and R is not the correct explanation of A

c) A is true, R is false

d) A is false, R is true

9. The solution of RC free response $\frac{dv_c}{dt}+\frac{1}{\tau}v_c=0$ with initial condition $v_c(0)=v_0$ is CO5 [K4]

a) $v_0 e^{t/\tau}$

b) $v_0 \sin(t/\tau)$

c) $v_0 e^{-t/\tau}$

d) $v_0 \sin(\tau t)$

10. Match list I and list II and select the correct answer using the codes given below CO2 [K2]

List I	List II
A. Gradient of a scalar point function ϕ is	i. $\vec{F}=\nabla\varphi$
B. Maximal directional derivative is	ii. 3
C. If a vector point function is irrorational, then	iii. $\lvert\nabla\phi\rvert$
D. The divergence of $\vec{r}$ is	iv. a vector

	A	B	C	D
a)	i	ii	iii	iv
b)	ii	iii	iv	i
c)	ii	i	iv	iii
d)	iv	iii	i	ii

PART B (10 x 2 = 20 Marks)
(Answer not more than 40 words)

11. By changing into polar coordinates, evaluate $\int_0^a\int_y^a \frac{x}{x^2+y^2}\,dxdy.$ CO1 [K2]

12. Evaluate $\int_0^1\int_0^{1-x}\int_0^{x+y} x\,dzdydx.$ CO1 [K4]

13. Determine the constants a,b,c so that the vector given below is irrorational CO2 [K2]
$\vec{F}=(x+2y+az)\vec{i}+(bx-3y-z)\vec{j}+(4x+cy+2z)\vec{k}$

14. Using Green's theorem, find the area of circle. CO2 [K3]

15. Test the analyticity of the function $f(z)=e^{-z}$. CO3 [K5]

16. Identify the fixed points of the transformation $w=\frac{2z-5}{z+4}$. CO3 [K3]

17. Expand $f(z)=\cos z$ in a Taylor's series about $z=\frac{\pi}{4}$. CO4 [K2]

18. Find the residue of $f(z)=\frac{z+2}{(z-2)(z+1)^2}$ at $z=-1$. CO4 [K4]

19. Determine $L\left[te^{-t}\sin 2t\right]$. CO5 [$K_2$]

20. Using Convolution theorem, find $L^{-1}\left[\frac{1}{(s+1)(s+2)}\right]$. CO5 [$K_3$]

Answer any FIVE Questions
PART C (5 x 14 = 70 Marks)
(Answer not more than 300 words)

Q.No. 21 is Compulsory

21. Verify Stoke's theorem of the vector field $\vec{F} = xy\vec{i} - 2yz\vec{j} - zx\vec{k}$ where S is the open surface of the rectangular parallelopiped formed by the plane $x=0, x=1, y=0, y=2, z=0, z=3$ above the xoy plane. CO2 [K_4]

22. (i) Change the order of integration in $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\,dy\,dx$ and hence evaluate it. (7) CO1 [K_5]

(ii) Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. (7) CO1 [K_3]

23. (i) Prove that $e^x(x\cos y - y\sin y)$ can be the real part of an analytic function and determine its harmonic conjugate. (7) CO3 [K_4]

(ii) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = 2, i, -2$. (7) CO3 [K_3]

24. (i) Make use of Cauchy's integral formula to evaluate $\int_c \frac{z+4}{z^2+2z+5}\,dz$, where c is the circle $|z+1+i| = 2$. (7) CO4 [K_5]

(ii) Evaluate $\int_0^{2\pi} \frac{d\theta}{13+5\sin\theta}$. (7) CO4 [$K_4$]

25. (i) Determine the Laplace transform of the triangular wave function given below: (7) CO5 [K_5]

$$f(t) = \begin{cases} t & 0 < t < b \\ 2b - t & b < t < 2b \end{cases} \text{ with } f(t) = (2b+t).$$

(ii) Solve the second order differential equation $y'' + 2y' + y = te^{-t}$ with initial conditions y(0) = 1 and $y'(0)$ = -2 by using Laplace transform. (7) CO5 [K_4]

26. (i) Determine the volume of the tetrahedron bounded by the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and the coordinate plane. (7) CO1 [K_4]

(ii) Expand $\frac{7z-2}{z(z-2)(z+1)}$ in the Laurent's series valid in $1 < |z+1| < 3$. (7) CO4 [K_5]

27. (i) Verify Gauss divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ taken over the cube $x=0, x=1, y=0, y=1, z=0, z=1$. (10) CO2 [$K_5$]

(ii) Identify the image of the infinite strip $\frac{1}{4} < y < \frac{1}{2}$ under the transformation $w = \frac{1}{z}$. (4) CO4 [K_3]

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