



M.TECH DEGREE EXAMINATIONS: MAY 2016

(Regulation 2015)

Second Semester

BIOTECHNOLOGY

P15MAT201: Applied Mathematics for Biotechnologists

COURSE OUTCOMES

- CO1:** Form and solve the ordinary differential equations of certain types
- CO2:** Acquire the knowledge in Laplace transforms and its properties.
- CO3:** Discover the equations of curve fit and compute various statistical measures
- CO4:** Analyze sample data and interpret the same for population.
- CO5:** Analyze the experimental design based on one-way, two-way and Latin squares

Time: Three Hours

Maximum Marks: 100

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. Examine the two statements carefully and select the answer using the codes given CO1 [K₂]
below:

Assertion (A) : The number of arbitrary constants present in the general solution of

$$\frac{d^2y}{dx^2} = y \text{ is } 2.$$

Reason (R) : The number of arbitrary constants present in the general solution of nth order differential equations is n+1.

- a) both A and R are individually true and R is the correct explanation of A
- b) both A and R are individually true but R is not the correct explanation of A
- c) A is true but R is false
- d) A is false but R is true.

2. Match list I with list II and select the correct answer using the codes given below. CO1 [K₃]

List I(ODE)	List II(P.I)
A. $(D^2 - 2D + 1)y = \cosh x$	i. $\frac{e^{2x}}{7}$
B. $(D^3 - 1)y = e^{2x}$	ii. $\frac{x^2}{4}e^x + \frac{e^{-x}}{8}$
C. $(D^2 + 1)y = x^3$	iii. $\frac{x^2}{4}e^x - \frac{e^{-x}}{8}$
D. $(D^2 - 2D + 1)y = \sinh x$	iv. $x^3 - 6x$

Codes:

	A	B	C	D
a)	ii	i	iv	iii
b)	iii	i	iv	ii
c)	i	ii	iv	iii
d)	ii	i	iii	iv

3. Consider the following Statements:

CO2 [K₂]

- 1) $L(1) = \frac{1}{s}$
- 2) $L(\cosh 2t) = \frac{2}{s^2 - 4}$
- 3) $L(e^{at}) = \frac{1}{s - a}$
- 4) $L(t^n) = \frac{n!}{s^n}$

Which of the above statements are correct?

- | | |
|--------|------------|
| a) 1,2 | b) 1,3 |
| c) 3,4 | d) 1,2,3,4 |

4. Match list I with list II and select the correct answer using the codes given below.

CO2 [K₃]

List I	List II
A. $L^{-1}\left(\frac{1}{s(s+1)}\right)$	i. $1 - e^{-t}$
B. $L^{-1}\left(\frac{1}{s(s+2)}\right)$	ii. $(1 - e^{-2t})\frac{1}{2}$
C. $L^{-1}\left(\frac{1}{s(s-1)}\right)$	iii. $e^t - 1$
D. $L^{-1}\left(\frac{1}{s(s-2)}\right)$	iv. $\frac{1}{2}(e^{2t} - 1)$

	A	B	C	D
a)	i	ii	iii	iv
b)	ii	i	iv	iii
c)	i	iii	ii	iv
d)	iii	iv	i	ii

5. Assertion (A): For a given set of x and y values, it is possible to have - 0.79 and 0.8 as the regression coefficients CO3 [K₂]

Reason (R): Both the regression coefficients should have the same sign

- | | |
|---|---|
| a) both A and R are individually true and R is the correct explanation of A | b) both A and R are individually true but R is not the correct explanation of A |
|---|---|

13. If $f(t) = e^{-2t} \sin 2t$ then find $L[f(t)]$. CO2 [K₃]
14. State the conditions of existence of Laplace transforms. CO2 [K₂]
15. Write down the normal equations to fit a quadratic curve by least square method. CO3 [K₁]
16. If the marks of 5 students are 45, 92, 26, 81 and 72, find the range and coefficient of range. CO3 [K₂]
17. What do you mean by critical region and acceptance region CO4 [K₁]
18. Write the applications of the t – distribution. CO4 [K₁]
19. What are the three basic principles of design of experiments? CO5 [K₁]
20. Is a 2 x 2 Latin Square possible? Explain. CO5 [K₂]

PART C (10 x 5 = 50 Marks)

21. Solve $(D^2 - 4D + 4)y = e^{2x}x^4 + \cos 2x$. CO1 [K₃]
22. Determine the Laplace transform of (i) $te^{-3t} \sin t$ (ii) $\frac{1-e^t}{t}$ CO2 [K₄]
23. Obtain the Laplace Transform of the “triangular wave” function $f(t)$ is defined as CO2 [K₄]
- $$f(t) = \begin{cases} t, & 0 < t < a \\ 2a-t, & a < t < 2a \end{cases} \text{ where } f(t+2a) = f(t)$$
24. Apply Convolution Theorem to find the inverse Laplace Transform of $\frac{1}{s^3(s+5)}$. CO2 [K₃]
25. By the method of least squares frame the best fitting straight line to the data given CO3 [K₄]
- below:

x:	5	10	15	20	25
y:	15	19	23	26	30

26. Petroleum pollution in oceans is known to increase the growth of a certain bacteria. CO3 [K₄]
- Brian did a project for his ecology class for which he made a bacteria count(per 100 milliliters) in nine random samples of sea water. His counts gave the following reading:
- 17 23 18 19 21 16 12 15 18
- (i) Find the sample mean
- (ii) Determine the sample standard deviation.

27. From the following data: CO3 [K₃]

X	45	64	37	58	41	29
Y	32	46	25	44	32	18

Calculate coefficient of correlation for the data.

28. A company claims that its light bulbs are superior to those of its main competitor. If a study showed that a sample of $n_1=40$ of its bulbs has a mean lifetime of 647 hours of continuous use with a standard deviation of 27 hours, while a sample of $n_2=40$ bulbs made by its main competitor had a mean lifetime of 638 hours of continuous use with a standard deviation of 31 hours, does this substantiate the claim at the 0.05 level of significance?.

CO4 [K₃]

29. Consider the following measurements of the heat producing capacity of the coal produced by two mines:

CO4 [K₄]

Mine 1:	8260	8130	8350	8070	8340	
Mine 2:	7950	7890	7900	8140	7920	7840

Can it be concluded that the two population variances are equal?

30. The quantity of dissolved oxygen is a measure of water pollution in lakes, rivers, and streams. Water samples were taken at four different locations in a river in an effort to determine if water pollution varied from location to location. Location I was 500 m above an industrial plant water discharge and near the shore. Location II was 200 m above the discharge point and in midstream. Location III was 50 m downstream from the discharge point and in midstream. Location IV was 200 m downstream from the discharge point and in midstream. The following table shows the results. Lower dissolved oxygen readings mean more pollution. Because of the difficulty in getting midstream samples, ecology student's collecting the data had fewer of these samples. Obtain the sum of squares between locations.

CO5 [K₄]

Location I	Location II	Location III	Location IV
7.3	6.6	4.2	4.4
6.9	7.1	5.9	5.1
7.5	7.7	4.9	6.2
6.8	8.0	5.1	
6.2		4.5	

PART D (2 x 10 = 20 Marks)

31. Solve: $\frac{dx}{dt} + y = \sin t; x + \frac{dy}{dt} = \cos t$ given that $x = 2$ and $y = 0$ at $t = 0$.

CO1 [K₃]

32. A certain injection administered to each of 12 patients resulted in the following increases of blood pressure: CO4 [K₄]

5,2,8,-1,3,0,6,-2,1,5,0,4

Can it be concluded that the injection will be, accompanied by an increase in blood pressure.

33. An experiment was designed to study the performance of 4 different detergents for cleaning fuel injectors. The following “cleanness” reading were obtained with specially designed equipment for 12 tanks of gas distributed over 3 different models of engines: CO5 [K₅]

	Engine 1	Engine 2	Engine 3
Detergent A	45	43	51
Detergent B	47	46	52
Detergent C	48	50	55
Detergent D	42	37	49

Looking on the detergents as treatments and the engines as blocks, obtain the appropriate analysis of variance table and test at the 0.01 level of significance whether there are differences in the detergents or in the engines.
