Register Number:……………..

KUMARAGURU college of technology character is life

B.E/B.TECH DEGREE EXAMINATIONS: NOV/DEC 2016

(Regulation 2014)

Fifth Semester

U14MAT506 : PROBABILITY AND QUEUEING THEORY

(Common to CSE & IT)

(Use of Statistical tables permitted)

COURSE OUTCOMES

CO1: Analyze random or unpredictable experiments and investigate important features of random experiments

CO2: Construct probabilistic models for observed phenomena through distributions which play an important role in many engineering applications

CO3: Associate random variables by designing joint distributions and correlate the random variables

CO4: Know about random processes, in particular about Markov chains which have applications in engineering

CO5: Identify the queueing model in the given system, find the performance measures and analyse the result

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. If A and B are any two events, then CO1 [K_1]

   a) $P(A\cup B)=P(A)+P(B)+P(A\cap B)$ b) $P(A\cup B)=P(A)+P(B)$

   c) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ d) $P(A\cup B)=P(A)-P(B)$

2. Consider the following statements CO2 [K_2]

   1. If A is an event, then $0\le P(A)\le 1$, i.e. the probability of an event lies between 0 and 1
   2. If $P(A)=1$, then the event A is called impossible event
   3. If A and B are dependent events, then the conditional probability of A given B is $P(A/B)=\dfrac{P(A\cap B)}{P(B)}$ if $P(B)\neq 0$
   4. The probability of the complement event $\overline{A}$ of A is $P(\overline{A})=1-P(A)$

   Which of the above statements are correct?

   a) 1, 2, 3 b) 1, 3, 4

   c) 2, 3, 4 d) 1, 2, 4

3. If X is a random variable and 'a' is a constant, then $\text{Var}\,(aX)$ is equal to CO3 [K_1]

   a) $a\,\text{Var}\,(X)$ b) $\dfrac{1}{a^2}\text{Var}\,(X)$

   c) $\dfrac{1}{a}\text{Var}\,(X)$ d) $a^2\,\text{Var}\,(X)$

4. The mean and variance of exponential distribution is CO3 [K_1]

   a) λ and λ b) λ and λ^2

   c) $\dfrac{1}{\lambda}$ and $\dfrac{1}{\lambda}$ d) $\dfrac{1}{\lambda}$ and $\dfrac{1}{\lambda^2}$

5. The random process is called continuous random sequence if CO4 [K_2]

a) State space S is continuous and index set T is discrete
b) State space S is discrete and index set T is continuous
c) Both state space S and index set T are continuous
d) Both state space S and index set T are discrete

6. Write the sequence of steps to prove that the states of a Markov chain are ergodic CO4 [K_2]

1) Verify that all the states have period 1 and that the chain is finite and irreducible.
2) Write the transition probability matrix P
3) Hence all the states are aperiodic and non-null persistent.
4) Find P^2, P^3,……

a) 2-4-3-1
b) 1-2-3-4
c) 2-4-1-3
d) 4-2-3-1

7. Formula of finding probability that the waiting time of a customer in the system exceeds ‘t’ CO5 [K_1]

a) $e^{-(\mu+\lambda)t}$
b) $e^{-(\lambda+\mu)t}$
c) $e^{-(\mu-\lambda)t}$
d) $e^{-(\lambda-\mu)t}$

8. Assertion (A): ‘Effective arrival’ is applicable for finite capacity Markovian queueing models CO5 [K_1]
Reason (R): ‘Effective arrival’ is a combination of ‘arrival’ and ‘getting service’

a) A is false but R is true
b) Both A and R are individually true and R is the correct explanation of A
c) A is true but R is false
d) Both A and R are individually true and R is not the correct explanation of A

9. A queueing system in which new customers __________ enter and existing ones _________ depart is called closed Jackson network. CO5 [K_1]

a) may, never
b) never, may
c) may, may
d) never, never

10. If 0,1 and b denote the free, busy and blocked states respectively and (i, j) represent the states of station 1 and station 2 respectively, then the possible states of the system are CO5 [K_2]

List I (i, j)		**List II (Description)**	
A.	0,0	1.	System is blocked
B.	1,1	2.	System is empty
C.	b,1	3.	Customer in Station 2 only
D.	0,1	4.	Customers in both station

	A	B	C	D
a)	1	4	3	2
b)	1	2	3	4
c)	2	4	1	3
d)	2	4	3	1

PART B (10 x 2 = 20 Marks)
(Answer not more than 40 words)

11. If at least one child in a family of three children is a boy, what is the probability that all are boys? CO1 [K_2]

12. A bag contains 5 white and 3 black balls. Two balls are drawn at random one after the other without replacement. Find the probability that the balls drawn are black. CO1 [K_3]

13.	A book of 500 pages contains 500 mistakes. Find the probability that there are at least four mistakes in a randomly selected page.	CO2	[K_2]
14.	State and prove memoryless property of exponential distribution	CO3	[K_2]
15.	Define Markov Process	CO4	[K_2]
16.	Consider the random process $\{X(t)\},\ X(t)=\cos(t+\phi)$, where is ϕ uniform in $\left(-\frac{\pi}{2},\ \frac{\pi}{2}\right)$. Check whether the process is stationary.	CO4	[K_2]
17.	What do the letters in the symbolic representation (a / b / c) : (d / e) of a queueing model represent?	CO5	[K_1]
18.	People arrive to purchase cinema tickets at the average rate of 6 per minute, and it takes an average of 7.5 seconds to purchase a ticket. If a person arrives 2 minutes before the picture starts and it takes exactly 1.5 minutes to reach the correct seat after purchasing the ticket, can he expect to be seated for the start of the picture?	CO5	[K_2]
19.	Write Pollaczek-Khintchine formula for Non-Markovian queueing model	CO5	[K_1]
20.	State the arrival theorem for closed Jackson network	CO5	[K_1]

Answer any FIVE Questions:-

PART C (5 x 14 = 70 Marks)

(Answer not more than 300 words)

Q.No. 21 is Compulsory

21. i) The chances of three candidates A, B and C becoming the manager of a company are in the ratio 3: 5: 4. The probability that a special bonus scheme will be introduced by them if selected are 0.6, 0.4 and 0.5 respectively. If the bonus scheme is introduced, what is the probability that B has become the manager? (7) CO1 [K_3]

ii) A random variable X has the following probability function (7) CO2 [K_3]

X =x	0	1	2	3	4	5	6	7	8
P(x)	a	3a	5a	7a	9a	11a	13a	15a	17a

a) Determine the value of ‘a’

b) Find the cumulative distribution function of X.

c) Find the smallest value of x for which F(x) is greater than 0.5

d) Find the largest value of x for which F(x) is less than 0.25

22. i) Derive the moment generating function of Binomial distribution and hence find its mean and variance. (7) CO2 [K_2]

ii) The mean yield for one acre plot is 662 kilos with a standard deviation of 32 kilos. Assuming normal distribution, how many 1 acre plots in a batch of 1000 plots would you expect to have (7) CO2 [K_3]

Over 700 kilos

Below 650 kilos

What is the lowest yield of best 100 plots?

23. i) If X and Y are two random variables having joint density function (7) CO3 [K_3]

$$f(x,y)=\begin{cases}\frac{1}{8}(6-x-y), & 0<x<2,\ 2<y<4,\\ 0, & \text{Otherwise}\end{cases}$$

a) Find $P(X<1\cap Y<3)$

b) Find $P(X<1/Y<3)$

c) Find $P(X+Y<3)$.

ii) Marks obtained by 10 students in Mathematics (X) and Statistics (Y) are given by (7) CO3 [K_2]

X	60	34	40	50	45	40	22	43	42	64
Y	75	32	33	40	45	33	12	30	34	51

Find the two regression lines.

24. i) Show that the random process $X(t) = A\cos(\omega_0 t + \theta)$ is wide sense stationary, if A and ω_0 are constants and θ is uniformly distributed random variable in $(0, 2\pi)$. (7) CO4 [K_3]

ii) A man goes to his office by bus or catches an auto every day. He never goes two days in a row by auto, but if he goes by bus one day, then the next day he is just as likely to go by bus as he is to travel by auto. Now suppose that on the first day of the week, the man tossed a fair dice and went by bus to work if and only if a '6' appeared.
Find the probability that he went by auto on the third day
Find the probability that he went by bus to work in long run. (7) CO4 [K_4]

25. i) There are 3 typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour, (7) CO5 [K_4]

a) What fraction of time will all the typists be busy?
b) What is the average number of letters waiting to be typed?
c) What is the average time a letter has to spend for waiting and being typed?

ii) Consider a system with two servers where customers arrive from outside the system in a Poisson fashion at server 1 at a rate of 4/hour and at server 2 at a rate of 5/hour. The customers are served at station 1 and station 2 at the rate of 8/hour and 10/hour respectively. A customer, after completion of service at server 1 is equally likely to go to server 2 or to leave the system. A customer departing from server 2 will go 25 percent of the time to server 1 and will depart from the system otherwise. Determine (7) CO6 [K_4]
(i) The total arrival rates at server 1 and server 2
(ii)Expected number of customers in the system

26. i) The arrivals at the counter in a bank occur in accordance with a Poisson process at an average rate of 8 per hour. The duration of service of a customer has an exponential distribution with a mean of 6 minutes. Find (7) CO5 [K_3]
a) the probability that an arriving customer has to wait
b) the average number of customers in the system
c) the average waiting time in the queue
d) the probability that there are 4 customers in the system.

ii) A car servicing station has 2 bays where servicing can be offered simultaneously. Due to space limitation, only four cars are accepted for servicing. The arrival pattern is Poisson with 12 cars per day. The service time in both the bays is exponentially distributed with $\mu = 8$ cars per day per bay. Find the average number of cars waiting to be serviced. (7) CO5 [K_4]

27. An automatic car wash facility operates with only one bay. Cars arrive according to a Poisson distribution with a mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. Find L_s, L_q, W_s, W_q, if the service time CO5 [K_4]

a) is constant and equal to 10 minutes
b) follows uniform distribution between 8 and 12 minutes
c) follows normal distribution with mean 12 minutes and S.D. 3 minutes
d) follows a discrete distribution with values 4, 8 and 15 minutes with corresponding probabilities 0.2, 0.6 and 0.2.

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