KUMARAGURU
college of technology
character is life

B.TECH DEGREE EXAMINATIONS: MAY 2017

(Regulation 2015)

Fourth Semester

INFORMATION TECHNOLOGY

U15MAT403: Discrete Mathematics

COURSE OUTCOMES

CO1: Have a better understanding of sets and application of set theory.
CO2: Acquire the knowledge of relations, equivalence relations and their properties.
CO3: Understand different kinds of functions.
CO4: Understand logical arguments and constructs simple mathematical proofs.
CO5: Know various graphs and learn different algorithms.
CO6: Acquire the knowledge of partially ordered sets, lattices, Boolean algebra and able to apply in circuits.

Time: Three Hours   **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. Match the following: For the two sets A and B,   CO1 [K1]

List I	List II
A. $A-(A-B)$	i. $(A \cap B) \cup (B \cup \phi)$
B. $(A \cap B) \cup (A \cap \overline{B})$	ii. $A^c \cap B$
C. The dual of $(A \cup B) \cap (B \cap U)$ is	iii. $A \cap B$
D. $B-A$	iv. A

   a)   A-(i),B-(iv),C-(iii),D-(ii)   c)   A-(iii),B-(iv),C-(i),D-(ii)

   b)   A-(ii),B-(iv),C-(i),D-(iii)   d)   A-(iii),B-(ii),C-(iv),D-(i)

2. The contrapositive of $Q \rightarrow P$ is   CO4 [K₂]

   a) $\neg Q \rightarrow \neg P$ b) $\neg P \rightarrow \neg Q$

   c) $Q \rightarrow P$ d) $Q \leftrightarrow P$

3. Which of the following statements are true: CO5 [K1]

   1. Regular graph is a graph in which all the vertices are of equal degree.

   2. The number of odd degree vertices in a graph G is always even.

   3. A null graph of more than one vertex is connected graph.

   4. In a complete graph of 'n' vertices, the degree of every vertex is two.

   a)   1,3   b)   1,4

   c)   1,2   d)   2,3

4. If $A = \{2,3,4,5\}, B = \{x : x \in N \text{ is an even number}\}$ then

      CO1 [K1]

    a) A is a subset of B   b) B is a subset of A

    c) A is not a subset of B   d) B is equivalent to A

5. Assertion (A): The negation of $(\forall x \in U)(x < 25)$ is $(\forall x \in U)(x \geq 25)$.

      CO4 [K2]

    Reason (R): If $A \Rightarrow B$ then A is called antecedent.

    a) Both A and R are Individually true and   b) Both A and R are Individually true but
       R is the correct explanation of A   R is not the correct explanation of A

    c) A is true but R is false   d) A is false but R is true

6. Given relations R and S, write the correct sequence of steps to verify that $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$, using matrices.

      CO2 [K2]

    1. Find $M_{R^{-1}}, M_{S^{-1}}, M_{R \circ S}$

    2. Verify $M_{(R \circ S)^{-1}} = M_{S^{-1} \circ R^{-1}}$

    3. Write the relation matrices M_R, M_S.

    4. Determine $M_{(R \circ S)^{-1}}$, $M_{S^{-1} \circ R^{-1}}$.

    a) 1-4-3-2   b) 3-2-4-1
       3-1-4-2   d) 1-3-4-2

7. Arrange in a sequence to find $f \circ g \circ h$ where $f(x) = x - 1, g(x) = 3x, h(x) = \begin{cases} 0 & \text{if } x \text{ is even} \\ 1 & \text{if } x \text{ is odd} \end{cases}$

      CO3 [K2]

    1. $(f \circ g \circ h)(x) = (f(g \circ h))(x)$

    2. $(g \circ h)(x) = \begin{cases} 0 & \text{if } x \text{ is even} \\ 3 & \text{if } x \text{ is odd} \end{cases} = v(x)$ (say)

    3. $(f \circ g \circ h)(x) = (f \circ v)(x)$

    4. $(f \circ g \circ h)(x) = \begin{cases} -1 & \text{if } x \text{ is even} \\ 2 & \text{if } x \text{ is odd} \end{cases}$

    a) 1-2-3-4   b) 1-3-2-4
    c) 2-4-1-3   d) 3-4-1-2

8. Match the following:

      CO3 [K1]

List I	List II
A. $f : X \to Y$ defined by $f(x) = x, \forall x \in X$	i. Composition of functions
B. $(g \circ f)(x) = g[f(x)]$	ii. constant function
C. $f : X \to Y$ if $f(x) = c, \forall x \in X$	iii. f is one to one
D. $f : Z \to Z$ defined by $f(x) = x + 1, \forall x \in Z$	iv. identity function

    a) A-(i),B-(iv),C-(iii),D-(ii)   b) A-(ii),B-(iv),C-(i),D-(iii)
    c) A-(iv),B-(i),C-(ii),D-(iii)   d) A-(ii),B-(iv),C-(iii),D-(i)

9. Assertion (A): There cannot be a Boolean algebra with 6 elements.

      CO6 [K2]

    Reason (R): The number of elements in a Boolean algebra is 2^n, for a natural number n.

    a) Both A and R are Individually true and   b) Both A and R are Individually true but
       R is the correct explanation of A   R is not the correct explanation of A

    c) A is true but R is false   d) A is false but R is true

10. Let $(L,*,\oplus,',0,1))$ be a Boolean algebra then $(a \oplus b)'$ is   CO6 [K1]

   a) $(a \oplus b)$ b) $(a' * b')'$

   c) $(a' \oplus b')$ d) $(a' * b')$

PART B (10 x 2 = 20 Marks)
(Answer not more than 40 words)

11. In a class 40% of the students enrolled for Mathematics and 70% enrolled for Economics. If   CO1 [K3]
    15% of the students enrolled for both Mathematics and Economics, what % of the students of
    the class did not enroll for either of the two subjects?

12. Give an example of a relation which is symmetric, transitive but not reflexive on {a,b,c}.   CO2 [K2]

13. Let X = {1,2,3,4}. Let $R = \{(x,y)/x, y \in X \text{ and } x - y \text{ is a multiple of } 2\}$.   CO2 [K2]

    $S = \{(x,y)/x, y \in X \text{ and } x - y \text{ is a multiple of } 3\}$. Find $R \cap S, R \cup S$.

14. If f and g are functions from the set of integers to the set of integers defined by   CO3 [K2]

    $f(x) = 2x + 3 \,\&\, g(x) = 3x + 2$. Find $f \circ g$.

15. Verify that $((P \vee Q) \wedge \neg P) \to Q$ is a tautology.   CO4 [K2]

16. Express the statement in symbolic form: "The crop will be destroyed if there is a flood".   CO4 [K2]

17. Define Planar graph.   CO5 [K1]

18. Draw the complete bipartite graphs $K_{2,3}$ and $K_{3,3}$.   CO5 [K2]

19. Let $A = \{1,2,3,4\}$ and let r be the relation $\leq$ on A. Draw the Hasse diagram of r.   CO6 [K₂]

20. Define Boolean Algebra and give example.   CO6 [K1]

Answer any FIVE Questions:-
PART C (5 x 14 = 70 Marks)
(Answer not more than 300 words)

Q.No. 21 is Compulsory

21. Using Dijikstra's algorithm, determine the shortest path between the vertices a to z as   (14)   CO5 [K4]
    shown below.
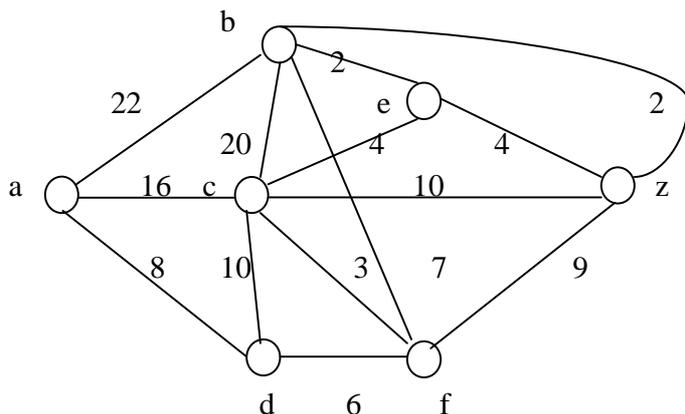
22. i) If $A = \{1,2,3\}, B = \{3,4,5\}, C = \{4,5,6\}$ then find   (7)   CO1 [K3]

    $A \times (B \cap C), (A \times B) \cup (A \times C), (A \times B) \cap (A \times C)$.

ii) Determine the number of integers between 1 and 250 that are not divisible by 2,3 or 5. (7) CO1 [K4]

23. (7) CO2 [K3]

a) Let R be a relation, with $M_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. Find the matrices that represent $i)\,R^2$

$ii)\,R^3 \quad iii)\,R^{-1} \quad iv)\,\overline{R}$

b) Let $A = R \times R$, where R = set of real numbers. The relation S on A is defined by $(a,b)S(c,d)$ if and only if $a^2 + b^2 = c^2 + d^2$. Prove that S is an equivalence relation. (7) CO2 [K3]

24. Let $f: R \to R\, \&\, g: R \to R$ where R is the set of real numbers given by $f(x) = x^2 - 2$ and $g(x) = x + 4$. Find $f \circ g\, \&\, g \circ f$. State whether these functions are injective, surjective & bijective. CO3 [K4]

25. a) Obtain principle conjunctive normal form and principle disjunctive normal form of $(P \to (Q \wedge R)) \wedge (\neg P \to (\neg Q \wedge \neg R))$. (7) CO4 [K5]

b) "If the band could not play the rock music or the refreshments were not delivered on time, then the new year's party would have been cancelled and Arun would have been angry". "If the party were cancelled, then refunds would have had to be made". "No refunds were made". Therefore, "the band could play rock music". Test the validity of above argument. (7) CO4 [K6]

26. a) Let $(L, \le)$ be a lattice, for any $a, b, c \in L$, then prove that $b \le c \Rightarrow a * b \le a * c, a \oplus b \le a \oplus c$. (7) CO6 [K4]

b) In a Boolean algebra, prove that $b \le c \Rightarrow a + b \le a + c$. (7) CO6 [K3]

27. Let A = {1,2,3,4} and R = {(1,2),(2,3),(3,4),(2,1)}. Using Warshall's algorithm find the transitive closure of R. CO2 [K3]

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