Register Number:……..

KUMARAGURU college of technology
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GENERAL INSTRUCTIONS TO THE CANDIDATES

1. Candidates are instructed to answer the questions as per Bloom's Taxonomy knowledge level (K_1 to K_6)
2. Candidates are strictly instructed not to write anything in the question paper other than their roll number.
3. Candidates should search their pockets, desks and benches and handover to the Hall Superintendent/ Invigilator if any paper, book or note which they may find therein as soon as they enter the examination hall.
4. Candidates are not permitted to bring electronic watches with memory, laptop computers, personal systems, walkie-talkie sets, paging devices, mobile phones, cameras, recording systems or any other gadget / device / object that would be of unfair assistance to him / her.
5. Corrective measures as per KCT examination policies will be imposed for malpractice in the hall like copying from any papers, books or notes and attempting to elicit the answer from neighbours.

B.E/B.TECH DEGREE EXAMINATIONS: MAY / JUNE 2017

(Regulation 2015)

Second Semester

U15MAT201: ENGINEERING MATHEMATICS - II

Common to All Branches

COURSE OUTCOMES

CO1: Evaluate double integral and triple integral to compute area, volume for two dimensional and three dimensional solid structure.

CO2: Know the gradient, divergence and curl, related theorems useful for engineering applications.

CO3: Test the analyticity and construct the analytic function and transform complex functions from one plane to another plane graphically

CO4: Evaluate real and complex integrals over suitable closed paths or contours.

CO5: Know the applications of Laplace transform and its properties and solve certain linear differential equations using Laplace transform technique.

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. Which of the following statements are true? CO1 [K_3]

   1. $\int_1^2\int_2^3 (x^2+2x+2)dxdy = \int_2^3\int_1^2 (x^2+2x+2)dydx$.
   2. $\int_1^2\int_2^3\int_2^4 dxdydz$ represents the volume of a cube.
   3. $\int_0^1\int_0^x dxdy$ represents the area of a triangle.
   4. $\iint_R f(x,y)dxdy = \iint_R f(r\cos\theta, r\sin\theta)rdrd\theta$

   a) 1,2,3 b) 1,2,4
   c) 1,3,4 d) 2,3,4

2. $\int_0^1\int_0^{\sqrt{1-x^2}} dxdy =$ CO1 [K_3]

   a) π b) 2π
   c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$

3. Write the correct sequence of steps to find the angle between the surfaces $\phi(x,y,z)=d$ and $\psi(x,y,z)=d'$ at the point (a,b,c) CO2 [K_3]

1. Determine the value of $\alpha=\cos\theta=\dfrac{\overline{n_1}.\overline{n_2}}{|\overline{n_1}|.|\overline{n_2}|}$
2. Find $\nabla\phi$ and $\nabla\psi$.
3. Evaluate $(\nabla\phi)_{(a,b,c)}$ and $(\nabla\psi)_{(a,b,c)}$.
4. Find $\theta=\cos^{-1}(\alpha)$

a) 1-2-3-4 b) 2-3-1-4
c) 1-4-2-3 d) 2-3-4-1

4. State which of the following statements are true: CO2 [K_2]

1. Gauss divergence theorem gives a relationship between a surface integral and a volume integral.
2. Green's theorem is a special case of Stoke's theorem.
3. The scalar form of Divergence theorem is

$$\iint_S\left(Pdxdy+Qdydz+Rdzdx=\iiint_V\left(P_z+Q_x+R_y\right)dxdydz\right)$$

4. If $\overline{F}$ is an irrotational vector, then it is conservative.

a) 1,2 b) 3,4
c) 2,3,4 d) 1,2,3,4

5. Examine the two statements carefully and select the answer using the codes given below: CO3 [K_4]
Assertion: The function $z\bar{z}$ is not analytic at the origin
Reason: $z\bar{z}$ is differentiable only at $z=0$

a) Both A & R are individually true and R is the correct explanation of A
b) Both A & R are individually true but R is the not correct explanation of A
c) A is true, R is false
d) A is false, R is true

6. Match List I with List II and select the correct answer using the codes given below , where $w=f(z)$ represents a transformation: CO3 [K_2]

List I	List II
A. Cross ratio	i. A point at which a function is not analytic.
B. Critical point	ii. The image of a point is itself under $w=f(z)$
C. Invariant point	iii. Invariant under Mobius transformation
D. Singular point	iv. A point at which $w=f(z)$ is not conformal

a) A-ii, B-iv, B-iii, D-i b) A-ii, B-i, C-iii, D-iv
c) A-iii, B-iv, C-ii, D-i d) A-iii, B-i, C-ii, D-iv

7. Write the correct sequence of steps to evaluate $\int_C\dfrac{7z-1}{z^2-3z-4}dz$ using Cauchy's integral formula, where C is the ellipse $x^2+4y^2=4$. CO4 [K_3]

1. $\int_C\dfrac{7z-1}{z^2-3z-4}dz=\int_C\dfrac{\left(\frac{7z-1}{z-4}\right)}{(z+1)}dz=\int_C\dfrac{f(z)}{(z+1)}dz$ where $f(z)=\dfrac{7z-1}{z-4}$ is analytic inside C.
2. $\int_C\dfrac{7z-1}{z^2-3z-4}dz=2\pi if(-1)=\dfrac{16\pi i}{5}$.

3. The poles are $z=4$ and $z=-1$, but only $z=-1$ lies inside C.

4. Consider $\int_C \frac{7z-1}{z^2-3z-4}dz = \int_C \frac{7z-1}{(z-4)(z+1)}dz$

a) 4-1-3-2 b) 4-3-1-2

c) 1-3-4-2 d) 1-4-3-2

8. Examine the two statements carefully and select the answer using the codes given below: CO4 [K_4]

Assertion: $z=0$ is an essential singularity of $f(z)=\frac{\sin z}{z}$.

Reason: $\underset{z\to 0}{Lim}\left(\frac{\sin z}{z}\right)=1$.

a) Both A & R are individually true and R is the correct explanation of A

b) Both A & R are individually true but R is the not correct explanation of A

c) A is true, R is false

d) A is false, R is true

9. Match List I with List II and select the correct answer using the codes given below: CO5 [K_3]

List I : $F(s)$	**List II:** $L^{-1}(F(s))$
A. $\frac{1}{s^2-2s+2}$	i. $e^t \sin t$
B. $\frac{1}{s^2-2s+1}$	ii. te^t
C. $\frac{1}{s^2-3s+2}$	iii. $e^t(e^t-1)$
D. $\frac{1}{s^2-s}$	iv. e^t-1

a) A-i, B-ii, C-iii, D-iv

b) A-ii, B-i, C-iv, D-iii

c) A-i, B-iv, C-iii, D-ii

d) A-ii, B-iii, C-i, D-iv

10. If $L(x(t))=X(s)$, then final value theorem for Laplace transforms states that CO5 [K_1]

a) $\lim_{t\to\infty} x(t)=\lim_{s\to 0} sX(s)$

b) $\lim_{t\to\infty} x(t)=\lim_{s\to\infty} sX(s)$

c) $\lim_{t\to 0} x(t)=\lim_{s\to 0} sX(s)$

d) $\lim_{t\to 0} x(t)=\lim_{s\to\infty} sX(s)$

PART B (10 x 2 = 20 Marks)
(Answer not more than 40 words)

11. Shade the region of integration in $\int_0^1 \int_{x^2}^{2-x} xydydx$. CO1 [$K_3$]

12. Find the area of the circle $r=2\cos\theta$ by double integration. CO1 [K_3]

The temperature at a point (x,y,z) in space is given by $T(x,y,z)=x^2+y^2-z$. CO2 [K_3]

13. A mosquito located at (4,4,2) desires to fly in such direction that it gets cooled faster. Find the direction in which it should fly.

14. State (i) Green's theorem and (ii) Gauss' divergence theorem. CO2 [K_1]

15. Show that the function $w=z^2$ is analytic. CO3 [K_3]

16. Identify the image of the half - plane $y\geq\frac{1}{4}$ under the transformation $w=\frac{1}{z}$. CO3 [K_3]

17. Evaluate $\int_C \frac{z}{(z-2)(z-3)^3}dz$ where C is the circle $|z-4|=\frac{1}{2}$. CO4 [K_3]

18. Find the residue of $f(z)=\frac{z}{(z-1)^2(z+2)}$ at its double pole. CO4 [K_3]

19. Determine $L(\cos^2 t)+L(\sin^2 t)$. CO5 [K_2]

20. Find the response v_C in the time domain, given the governing equation for the voltage drop across the capacitor as $C\frac{dv_C}{dt}+\frac{v_C}{R}=0,\ v_C(0)=0.$ CO5 [K_4]

Answer any FIVE Questions:-
PART C (5 x 14 = 70 Marks)
(Answer not more than 300 words)

Q.No. 21 is Compulsory

21. (i) Transform into polar coordinates and evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dydx$ (7) CO1 [K_4]

Hence estimate the value of $\int_0^\infty e^{-x^2}dx$.

(ii) Prove that the function $u=x^3-3xy^2+3x^2-3y^2+1$ is harmonic. Construct the analytic function $f(z)$ whose real part is u and find the conjugate harmonic function v. (7) CO3 [K_4]

22. (i) Change the order of integration in $\int_0^2\int_x^2(x^2+y^2)dydx$ and hence evaluate it. (7) CO1 [K_4]

(ii) Determine the volume of the tetrahedron bounded by the planes $x=0, y=0, z=0,$ and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ (7) CO1 [K_3]

23. Verify Stoke's theorem for $(y-z+2)\bar{i}-(4+yz)\bar{j}-zx\bar{k}$ over the open surface of the cube formed by x=0, x=2, y=0, y=2, z=2 above the XY plane. CO2 [K_4]

24. (i) If f(z) is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2$ (7) CO3 [K_3]

(ii) Identify the bilinear transformation which maps the points $1,0,-1$ of the z- plane onto the points $0,-1,\infty$ respectively , in the w- plane. Find the image of the unit circle $|z|=1$ under this transformation. (7) CO3 [K_3]

25. (i) Find the Laurent's series expansion of $f(z)=\frac{1}{(z+1)(z+3)}$ valid in (a) $|z|<1$ (7) CO4 [K_4]

(b) $1<|z|<3$ (c) $|z|>3$.

(ii) Evaluate $\int_{-\infty}^{\infty}\frac{x^2}{(x^2+1)(x^2+4)}dx$ using contour integration. (7) CO4 [K_4]

26. (i) Find the Laplace Transform of the "triangular wave" function f(t) whose graph is (7) CO5 [K_4]

$f(t)=\begin{cases}t, & 0<t<a\\ 2a-t, & a<t<2a\end{cases}$ where $f(t+2a)=f(t)$.

(ii) Solve using Laplace transforms: $y''+3y'+2y=e^t,\ y(0)=0,\ y'(0)=1$ (7) CO5 [K_4]

27. (i) Find the constant a such that $\bar{F}=(y^2\cos x+z^3)\bar{i}+(2y\sin x-4)\bar{j}+axz^2\bar{k}$ is irrotational and find its scalar potential. (7) CO2 [K_4]

(ii) Using convolution theorem, determine $L^{-1}\left(\frac{s^2}{(s^2+1)(s^2+9)}\right)$ (7) CO5 [K_3]

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