



B.E/B.TECH DEGREE EXAMINATIONS: MAY 2017

(Regulation 2015)

Fourth Semester

U15MAT401 : NUMERICAL METHODS

(Common to CE/ME/MCE/TXT & FT)

COURSE OUTCOMES

- CO1:** Solve a set of algebraic equations representing steady state models formed in engineering Problems.
- CO2:** Fit smooth curves for the discrete data connected to each other or to use interpolation methods over these data tables.
- CO3:** Find the trend information from discrete data set through numerical differentiation.
- CO4:** Estimate integrals from discrete data through numerical methods.
- CO5:** Predict the system dynamic behaviour through solution of ODEs modeling the system.
- CO6:** Solve PDE models representing spatial and temporal variations in physical systems through numerical methods.

Time: Three Hours

Maximum Marks: 100

**Answer all the Questions:-
PART A (10 x 1 = 10 Marks)**

1. Match List I with that of List II

CO2 [K₂]

List I	List II
A. Newton's forward interpolation formula	i. Both equal and unequal intervals
B. For finding the value of y which is nearer to the ending value of the table, we use	ii. Newton's backward difference interpolation formula
C. If the interval of differencing is not constant we can use	iii. Equidistant intervals
D. We can use Lagrange's interpolation method for	iv. Newton's divided difference and Lagrange's interpolation

- | | A | B | C | D |
|----|-----|----|-----|----|
| a) | ii | i | iii | iv |
| b) | iii | iv | ii | i |
| c) | ii | iv | iii | i |
| d) | iii | ii | iv | i |

2. The order of convergence of Regula-falsi method is

CO1 [K₂]

- | | |
|----------|------------|
| a) 2.618 | b) 1.618 |
| c) 1.518 | d) - 1.618 |

3. Which of the following statements are correct CO1 [K₂]
- 1) Newton-Raphson method is also called Method of tangents.
 - 2) The sufficient condition for solving a system of equations by Gauss-Seidel method, is that the coefficient matrix should be Diagonally dominant.
 - 3) The equation $f(x) = 0$ will have atleast one real root between a & b if $f(a)$ & $f(b)$ are of opposite signs.
 - 4) In Gauss Elimination method the coefficient matrix is transformed into a diagonal matrix.
- a) 1,3 b) 2,4
c) 1,2,3 d) 2,3
4. The error in Trapezoidal rule is CO4 [K₂]
- a) $|E| < \frac{(b-a)h^2}{12} M$ b) $|E| > \frac{(b-a)h^2}{12} M$
c) $|E| < \frac{(b+a)h^2}{12} M$ d) $|E| > \frac{(b+a)h^2}{12} M$
5. Assertion (A): The Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is elliptic CO6 [K₂]
- Reason (R): Equation is elliptic if $B^2 - 4AC > 0$
- a) Both A and R are Individually true and R is the correct explanation of A b) Both A and R are Individually true but R is not the correct explanation of A
c) A is true but R is false d) A is false but R is true
6. If $h = 1, \Delta y_0 = 3.0244, \Delta^2 y_0 = -0.0003, \Delta^3 y_0 = 0$ then the value of $\left(\frac{dy}{dx}\right)_{x=x_0}$ is CO3 [K₂]
- a) 0.02455 b) 0.02275
c) -0.0003 d) 0.0232
7. The step involved in finding the straight line fit. CO2 [K₃]
1. Calculate the unknowns required.
 2. Write the general equation of a straight line.
 3. Substitute the unknowns to get the straight line fitting the data
 4. Write the corresponding normal equations.
- a) 2-3-4-1 b) 4-2-1-3
c) 2-4-1-3 d) 4-1-3-2
8. The improved Euler method is based on the averages of _____ CO5 [K₁]
- a) points b) slopes
c) tangents d) normal
9. Assertion (A): Milne's Predictor-Corrector method is multi step method. CO5 [K₁]
- Reason (R) : A Predictor formula is used to predict the value of y at x_{i+1} and a corrector formula is used to correct the error and to improve that value of y_{i+1} .
- a) Both A and R are Individually true and R is the correct explanation of A b) Both A and R are Individually true but R is not the correct explanation of A
c) A is true but R is false d) A is false but R is true

10. The Explicit formula to solve the wave equation $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$ is CO6 [K₁]

a) $U_{i,j+1} = U_{i-1,j} + U_{i+1,j} - U_{i,j-1}$

b) $u_{i,j+1} = \frac{1}{2} [u_{i+1,j} + u_{i-1,j}]$

c) $U_{i,j} = \frac{1}{4} [U_{i-1,j-1} + U_{i-1,j+1} + U_{i+1,j-1} + U_{i+1,j+1}]$

d) $u_{i,j} = \frac{1}{4} [u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1}]$

PART B (10 x 2 = 20 Marks)
(Answer not more than 40 words)

- | | |
|---|-----------------------|
| 11. State the condition for convergence of fixed point iteration method. | CO1 [K ₁] |
| 12. Distinguish between direct and iterative method to solve simultaneous equations. | CO1 [K ₂] |
| 13. Write down the normal equations to fit a parabolic curve by least square method. | CO2 [K ₂] |
| 14. Explain inverse interpolation. | CO2 [K ₂] |
| 15. What are the two types of errors involving in the numerical computation of derivatives? | CO3 [K ₃] |
| 16. Evaluate $\int_0^1 \int_0^1 e^{x+y} dx dy$ with h = k = 0.5. | CO4 [K ₃] |
| 17. What do you mean by single step method? Give examples. | CO5 [K ₂] |
| 18. Using Euler's method solve $y' = y + e^x$, $y(0) = 1$ for $y(0.2)$, taking h = 0.2 | CO5 [K ₃] |
| 19. Name the two methods and their formulas to solve one – dimensional heat equation. | CO6 [K ₃] |
| 20. Write the formula to solve Poisson equation. | CO6 [K ₂] |

Answer any FIVE Questions:-
PART C (5 x 14 = 70 Marks)
(Answer not more than 300 words)

Q.No. 21 is Compulsory

21. i) A jet fighter's position on an aircraft carrier's runway was timed during landing, (8) CO3 [K₃]
where y is the distance from the end of the carrier. Estimate the velocity and acceleration at $t = 1.1$ and $t = 1.6$.

t (sec)	1	1.1	1.2	1.3	1.4	1.5	1.6
y(m)	7.989	8.403	8.781	9.129	9.451	9.75	10.031

- ii) The table gives the velocity of a moving particle at time t seconds. Find the distance (6) CO4 [K₃]
covered by the particle in 12 seconds and also the acceleration at $t = 2$ seconds.

t (sec)	0	2	4	6	8	10	12
v	4	6	16	34	60	94	136

22. i) Find the dominant eigenvalue and the corresponding eigenvector of $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$. (8) CO1 [K₃]

ii) Determine a root of the equation $\cos x - x e^x = 0$ using Newton Raphson method. (6) CO1 [K₃]

23. i) Find the polynomial of the lowest possible degree which assumes the values 1245, 33, 5, 9 and 1335 at $x = -4, -1, 0, 2$ and 5 , using Newton's divided difference interpolation formula. Also determine the value of the polynomial at abscissa 1. (7) CO2 [K₃]

ii) Estimate the number of students who obtained marks between 40 and 45, from the given data (7) CO2 [K₂]

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of students	31	42	51	35	31

24. i) Using Improved Euler's method solve numerically the equation $y' = x + y, y(0) = 1$, by taking $h = 0.2$, upto $x = 1$. Analyse the results with exact solution. (7) CO5 [K₃]

ii) Using Runge_Kutta method of fourth order, find $y(0.8)$ correct to 4 decimal places if $y' = y - x^2$ given $y(0.6) = 1.7379$. (7) CO5 [K₃]

25. i) Find the solution of the heat equation $u_t = u_{xx}$ subject to the condition $u(x,0)=0, u(0,t) = 0$ and $u(1,t) = t$. Compute u for $t=1/8$ in one step using Crank-Nicolson method. (7) CO6 [K₂]

ii) Solve $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the squares mesh bounded by $x = 0; y = 0; x = 3; y = 3$ with $u = 0$ on the boundary and mesh length is 1 unit. (7) CO6 [K₃]

26. i) Get the solution for the given system using Gauss-Seidel method (7) CO1 [K₃]
 $10p + q + r + s = 21.09, p + 10q + r + s = 31.08, p + q + 10r + s = 41.07,$
 $p + q + r + 10s = 51.06.$

ii) The table gives the results of an observation: θ is observed temperature in degrees centigrade of a vessel of cooling water, t is the time in minutes from the beginning of observation. Find the approximate rate of cooling at $t = 3$ and $t = 3.5$. (7) CO3 [K₃]

t	1	3	5	7	9
θ	85.3	74.5	67.0	60.5	54.3

27. Solve $y' = x^2 + y^2 - 2$ using Milne's predictor – corrector formula for $x = 0.3, y(0) = 1$. Evaluate the values of y for $x = -0.1, 0.1$ and 0.2 using Taylor's series. CO5 [K₃]
